

Determining Individual Relative Share Contributions of Nonlinear Loads in Overall Distortions of AC Power Line Voltage and Current

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Abstract—The paper provides the assessment method of the voltage and current THD values of some AC power line in the presence of an arbitrary number of some higher-harmonics-producing nonlinear loads. The technique is generalized here to the case of the presence of some equivalent non-sinusoidal EMF and the parameters R and L , which are reflecting its non-ideal nature, and the compensating capacitance C . This approach is based on the method of the algebraization of the differential equations. The technique makes it possible not only to predict the quality of the supplying AC power under certain known analytical descriptions or measured waveforms of the nonlinear loads' currents and the EMF, but also to assess their individual relative share contributions in the overall distortion of power quality. The offered method and considered equations may be useful for researchers and practical engineers dealing with AC power network both at the design and monitoring stages.

Keywords—AC power line, nonlinear loads, method of differential equations algebraization, THD, n -order integrated signal harmonics factor, individual load relative share contribution in higher harmonics component

I. INTRODUCTION

The problem of a reciprocal effect of nonlinear loads of electric energy and power line on electromagnetic processes of each other is still topical despite a lot of regulations in the field [1]-[4]. Issues, related to nonlinear loads, including their individual impact to the decline in power quality, have been actively discussed in numerous papers throughout the world, see the examples in [5]-[14]. One can find that the modern mathematical methods and computerized tools of artificial intelligence can be now involved in solving problems of identifying the energy quality of electrical systems, in addition to impedance calculations and conventional mathematical methods, like Fourier transform and correlation analysis.

Among the matters, concerning higher harmonics, the analytically-grounded methods have been proposed and considered in publications of the NSTU (Novosibirsk) research laboratory of conversion systems energy optimization to define the contributions of loads and AC/DC mains in voltage quality change. Some new calculation and measurement techniques have been offered on the base of the developed authored general direct method of electric energy quality analysis, the method of the differential equations algebraization (ADE) [15]. The resulting integrated square-

law estimations of non-sinusoidal voltages and currents make it possible to calculate the total forms values of harmonics factors of voltages and currents.

In contrast to many sophisticated methods, ADE-based techniques are simple in calculation, since they do not use neither advanced spectral (harmonious) analysis nor mathematical tools for differential equations solving. The proposed techniques concern not only the contributions of loads and mains in change of voltage quality at a point of common coupling (PCC), that is the conventional and the most investigated issue, but also the contributions of the loads and the mains in change of the current's quality of the AC power line [16, 17].

The purpose of this paper is to generalize the equations of the factors of individual relative share contributions of any arbitrary number N of nonlinear loads in the overall harmonic distortions of the AC power line voltage and current for the case of an enhanced power line model.

II. ADE PROCEDURE APPLYING TO POWER LINE MODEL

The considered equivalent circuit contains an AC power line equivalent non-sinusoidal electromotive force (EMF) e , elements with parameters R and L , which reflect its non-ideal nature, and a compensating capacitance C , see Fig. 1. The current sources designate the currents $i_1, i_2, \dots, i_N$ of N nonlinear loads connected to the PCC. It is assumed here that no DC voltage and current components are present, i.e. the steady-state mode is being considered.

The differential equations, establishing the relations between the instantaneous values of the voltage u across the capacitor C (voltage at PCC), of the power supply EMF e and the current i , and of the nonlinear loads' currents are:

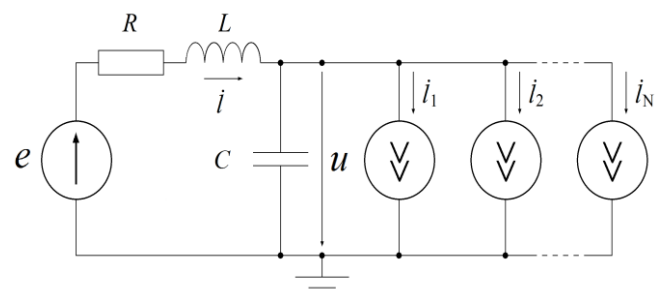


Fig. 1. Equivalent circuit of AC power line with nonlinear loads.

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$$L \frac{di}{dt} + Ri + u = e, \quad i = C \frac{du}{dt} + i_1 + i_2 + \dots + i_N. \quad (1)$$

Considering (1) for the higher harmonics components of the voltages e_h , u_h and currents i_{1h} , i_{2h} , ..., i_{Nh} , i_h leads to deducing the following normalized integral equations:

$$u_h + \frac{R}{L} u_h^{(-1)} + \frac{1}{LC} u_h^{(-2)} = \frac{1}{LC} e_h^{(-2)} -$$

$$-\frac{1}{C} \left(\tilde{i}_{1h}^{(1)} + \tilde{i}_{2h}^{(1)} + \dots + \tilde{i}_{Nh}^{(1)} \right) - \frac{R}{LC} \left(\tilde{i}_{1h}^{(2)} + \tilde{i}_{2h}^{(2)} + \dots + \tilde{i}_{Nh}^{(2)} \right), \quad (2)$$

$$i_h + \frac{R}{L} \tilde{i}_h^{(1)} + \frac{1}{LC} \tilde{i}_h^{(2)} = \frac{1}{L} e_h^{(-1)} + \frac{1}{LC} \left(\tilde{i}_{1h}^{(2)} + \tilde{i}_{2h}^{(2)} + \dots + \tilde{i}_{Nh}^{(2)} \right), \quad (3)$$

where $\tilde{x}_h^{(-n)}$ is the result of the n -fold indefinite integral taking of the instantaneous higher harmonics variable x_h .

In the context of the ADE technique [15], after rejection of the terms with integrated higher harmonics components of required variables (for the adoption of the first, i.e. the roughest level of approximation), applying the squaring and averaging operator $\frac{1}{T} \int_0^T (\dots)^2 dt$ (T is the period of the fundamental harmonic of the EMF e) we can obtain algebraic equations regarding to the RMS values of the voltage and current higher harmonics components:

$$U_h^2 = \frac{1}{(\omega C)^2} \sum_{p=1}^N \left(\overline{K}_{hip}^{(1)} \cdot I_{p(1)} \right)^2 + \left(\frac{R}{\omega^2 LC} \right)^2 \sum_{p=1}^N \left(\overline{K}_{hip}^{(2)} \cdot I_{p(1)} \right)^2 +$$

$$+ \frac{2}{C^2} \sum_{\substack{j,k=1 \\ k>j}}^N \left(\tilde{i}_{jh}^{(1)} \cdot \tilde{i}_{kh}^{(1)} \right) + 2 \left(\frac{R}{LC} \right)^2 \sum_{\substack{j,k=1 \\ k>j}}^N \left(\tilde{i}_{jh}^{(2)} \cdot \tilde{i}_{kh}^{(2)} \right) +$$

$$+ \left(\frac{\overline{K}_{he}^{(2)} \cdot E_{(1)}}{\omega^2 LC} \right)^2 - \frac{2}{LC^2} \sum_{s=1}^N \left(\tilde{i}_{sh}^{(1)} \cdot e_h^{(-2)} \right) - \frac{2R}{(LC)^2} \sum_{s=1}^N \left(\tilde{i}_{sh}^{(2)} \cdot e_h^{(-2)} \right), \quad (4)$$

$$I_h^2 = \frac{1}{(\omega^2 LC)^2} \sum_{p=1}^N \left(\overline{K}_{hip}^{(2)} \cdot I_{p(1)} \right)^2 + \frac{2}{(LC)^2} \sum_{\substack{j,k=1 \\ k>j}}^N \left(\tilde{i}_{jh}^{(2)} \cdot \tilde{i}_{kh}^{(2)} \right) +$$

$$+ \left(\frac{\overline{K}_{he}^{(1)} \cdot E_{(1)}}{\omega L} \right)^2 + \frac{2}{L^2 C} \sum_{s=1}^N \left(\tilde{i}_{sh}^{(2)} \cdot e_h^{(-1)} \right). \quad (5)$$

Here N is an arbitrary number of nonlinear loads; ω is the circular frequency of the fundamental harmonic of the voltage e , $\omega = 2\pi/T$; the following expressions with parentheses are the scalar products of the corresponding exposure variables:

$$\left(\tilde{x}_j^{(-m)}, \tilde{x}_k^{(-n)} \right) = \frac{1}{T} \int_0^T \tilde{x}_j^{(-m)} \tilde{x}_k^{(-n)} dt, \quad (6)$$

$\overline{K}_{hx}^{(-n)}$ is the n -order integrated signal harmonics factor (IFH) [15] of an exposure variable x ,

$$\overline{K}_{hx}^{(-n)} = \frac{\overline{X}_h^{(-n)}}{\overline{X}_{(1)}^{(-n)}} = \frac{\overline{X}_h^{(-n)} \cdot \omega^n}{X_{(1)}} = \sqrt{\sum_{k=2}^{\infty} \left(\frac{X_{(k)}}{k^n \cdot X_{(1)}} \right)^2}, \quad (7)$$

where $\overline{X}_{(1)}^{(-n)}$ and $\overline{X}_h^{(-n)}$ are the RMS values of the n times integrated fundamental $x_{(1)}$ and higher harmonics x_h components of an exposure x , $\tilde{x}_{(1)}^{(-n)}$ and $\tilde{x}_h^{(-n)}$, respectively; $X_{(k)}$ is the RMS value of k -order harmonic of x .

The various order IFHs reflect the load filtering effect on the investigated signal. They make weighted (by the harmonic order number) summation of harmonics, as a matter of fact, modeling the effect of the action of the amplitude-frequency characteristic of the idealized electric circuit of the corresponding order. Such the ideal filter reduces the magnitudes of every harmonic by the number of times equal to the harmonic order number raised to the filter order power. To calculate the IFH value for the exposure variables (the nonlinear loads' currents and the EMF), the known analytical descriptions or measured waveforms of them are needed.

As it can be seen from (5), obtained as the first level of approximation, the presence of parameter R has no practical impact on the RMS value of the current higher harmonics component.

Now it is possible to assess the voltage u and current i harmonics factors, i.e. the total harmonic distortion (THD) factors:

$$K_{hu} = \frac{U_h}{U_{(1)}}, \quad K_{hi} = \frac{I_h}{I_{(1)}}. \quad (8)$$

The RMS values $U_{(1)}$ and $I_{(1)}$ can be in an obvious way deduced from (2) and (3), written for the fundamental components of all the variables.

The independent significance of (4) and (5) lies in the possibilities for determining the individual load relative share contributions in higher harmonics components.

III. COEFFICIENTS OF PARTICIPATION OF INDIVIDUAL NONLINEAR LOADS IN OVERALL HARMONIC DISTORTION OF POWER QUALITY

The contributions of the exposure x (the EMF e , the nonlinear load i_k , $k \in \{1, 2, \dots, N\}$) to the square of the PCC voltage higher harmonics RMS component V_x and to the square of the power line current higher harmonics RMS component J_x are, respectively,

$$V_e = V_e^e + \sum_{k=1}^N V_e^{i_k}, \quad V_{i_k} = \sum_{s=1}^N V_{i_k}^{i_s} + V_{i_k}^e = V_{i_k}^{i_k} + \sum_{\substack{s=1 \\ s \neq k}}^N V_{i_k}^{i_s} + V_{i_k}^e, \quad (9)$$

$$J_e = J_e^e + \sum_{k=1}^N J_e^{i_k}, \quad J_{i_k} = \sum_{s=1}^N J_{i_k}^{i_s} + J_{i_k}^e = J_{i_k}^{i_k} + \sum_{\substack{s=1 \\ s \neq k}}^N J_{i_k}^{i_s} + J_{i_k}^e, \quad (10)$$

where V_x^x and J_x^x are the own contributions of the exposure x to U_h^2 and to I_h^2 correspondingly,

$V_{x_1}^{x_2}$ and $J_{x_1}^{x_2}$ are the shares of the joint contributions of the exposures x_1 and x_2 to U_h^2 and to I_h^2 in V_{x_1} and in J_{x_1} ,

$$V_{x_1}^{x_2} = V_{x_1, x_2} \frac{V_{x_1}^{x_1}}{V_{x_1}^{x_1} + V_{x_2}^{x_2}}, \quad J_{x_1}^{x_2} = J_{x_1, x_2} \frac{J_{x_1}^{x_1}}{J_{x_1}^{x_1} + J_{x_2}^{x_2}}, \quad (11)$$

V_{x_1, x_2} and J_{x_1, x_2} are the above mentioned joint contributions of the exposures x_1 and x_2 to U_h^2 and to I_h^2 . Obviously, $V_{x_1}^{x_2} + V_{x_2}^{x_1} = V_{x_1, x_2}$, $J_{x_1}^{x_2} + J_{x_2}^{x_1} = J_{x_1, x_2}$.

In particular, as we can see in (4),

$$\begin{aligned} V_{i_k}^{i_k} &= \left(\left(\overline{K}_{hik}^{(1)} \right)^2 + \left(R \frac{\overline{K}_{hik}^{(2)}}{\omega L} \right)^2 \right) \cdot \frac{I_{k(1)}^2}{(\omega C)^2}, \\ V_{i_k, i_s} &= \frac{2}{C^2} \cdot \left(\left(\overline{i}_{kh}^{(1)}, \overline{i}_{sh}^{(1)} \right) + \left(\frac{R}{L} \right)^2 \cdot \left(\overline{i}_{kh}^{(2)}, \overline{i}_{sh}^{(2)} \right) \right), \\ V_e^e &= \left(\frac{\overline{K}_{he}^{(2)}}{\omega^2 LC} \right)^2 \cdot E_{(1)}^2, \\ V_{e, i_k} &= -\frac{2}{LC^2} \cdot \left(\left(\overline{i}_{kh}^{(1)}, \overline{e}_h^{(1)} \right) + \frac{R}{L} \cdot \left(\overline{i}_{kh}^{(2)}, \overline{e}_h^{(2)} \right) \right); \\ V_{e, i_k}^{i_k} &= V_{e, i_k} \frac{V_e^e}{V_e^e + V_{i_k}^{i_k}}, \quad V_{i_k, i_s}^{i_k} = V_{i_k, i_s} \frac{V_{i_k}^{i_k}}{V_{i_k}^{i_k} + V_{i_s}^{i_s}}, \\ V_{i_k}^e &= V_{e, i_k} \frac{V_{i_k}^{i_k}}{V_e^e + V_{i_k}^{i_k}}. \end{aligned} \quad (12)$$

Similarly, in accordance with (5),

$$\begin{aligned} J_{i_k}^{i_k} &= \left(\frac{\overline{K}_{hik}^{(2)}}{\omega^2 LC} \right)^2 \cdot I_{k(1)}^2, \quad J_{i_k, i_s}^{i_k} = \frac{2}{(LC)^2} \cdot \left(\overline{i}_{kh}^{(2)}, \overline{i}_{sh}^{(2)} \right), \\ J_e^e &= \left(\overline{K}_{he}^{(1)} \right)^2 \cdot \frac{E_{(1)}^2}{(\omega L)^2}, \quad J_{e, i_k}^e = \frac{2}{L^2 C} \cdot \left(\overline{i}_{kh}^{(2)}, \overline{e}_h^{(2)} \right); \\ J_e^{i_k} &= J_{e, i_k} \frac{J_e^e}{J_e^e + J_{i_k}^{i_k}}, \quad J_{i_k, i_s}^{i_k} = J_{i_k, i_s} \frac{J_{i_k}^{i_k}}{J_{i_k}^{i_k} + J_{i_s}^{i_s}}, \\ J_{i_k}^e &= J_{e, i_k} \frac{J_{i_k}^{i_k}}{J_e^e + J_{i_k}^{i_k}}. \end{aligned} \quad (14)$$

The obtained expressions of the V_x and J_x components have a very clear and simple interpretation and can be easily calculated.

The corresponding to (9) and (10) relative share contributions (the individual coefficients of participation) of

the exposure x to the overall square of the PCC voltage higher harmonics RMS value $K_u(x)$ and to the overall square of the power line current higher harmonics RMS value $K_i(x)$ can be now defined:

$$K_u(x) = \frac{V_x}{U_h^2}, \quad K_i(x) = \frac{J_x}{I_h^2}. \quad (16)$$

Naturally, for the relative share contributions of all the exposures e and $i_1, i_2, \dots, i_N$ the following equations are true:

$$K_u(e) + \sum_{k=1}^N K_u(i_k) = 1, \quad K_i(e) + \sum_{k=1}^N K_i(i_k) = 1. \quad (17)$$

The experimental verification of the employed here baseline approach has been implemented through the computer simulations in PSIM [17]. The results for the three nonlinear loads (including a thyristor converter operating in various modes) have confirmed the efficiency of the proposed techniques.

In some cases the scalar products (6) of the harmonics-producing exposure variables in (4) and (5) possess negative values. This means that such kind of combinations of exposure variables corresponds to the mutual compensation of their undesirable higher harmonics components and leads to reducing the related overall harmonic distortion. It is also naturally reflected in reducing their both corresponding values in (9) or (10) (V_x or J_x) of the individual share contributions to the square of the respective higher harmonics RMS component.

The designed analytical background could have been a powerful tool for obtaining some new engineering solutions, a current monitoring of AC power line and carrying out some harmonics-compensating activities. In view of the clear and simple mathematical description, the developed technique of determining the individual relative share contributions of nonlinear loads in overall distortions of AC power line voltage and current should be taken into account to work out regulations and standards on the power quality and conductive type of electromagnetic compatibility.

IV. CONCLUSIONS

The assessment method of the voltage and current THD of AC power line with nonlinear loads is considered. The analytical expressions for the RMS values of the higher harmonics components are provided here for the generalized case of the presence of any arbitrary number N of nonlinear loads, some equivalent non-sinusoidal EMF, the parameters R and L , which are reflecting its non-ideal nature, and also the compensating capacitance C .

The technique, based on the method of the algebraization of the differential equations, has made it possible to define analytically the nonlinear loads individual relative share contributions to the higher harmonics components of the feeding network voltage and current.

The computer simulations have shown that the analytically-grounded compensation of a power line currents' components, generated by harmonics-producing loads, is promising and achievable with the knowledge of the

parameters of the above mentioned loads currents waveforms.

The offered method and provided equations may be useful for researchers and practical engineers dealing with AC power networks at the stages of their design (especially regarding to the distributed generation AC power systems), monitoring and harmonic distortion compensating.

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