

Weighted THD Factors Evaluation of Multilevel Voltage Source Inverter with Space Vector Control under Common-Mode Voltage Elimination

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Abstract—The harmonic distortion's characteristics of the nearest-vector-selecting space vector control with common-mode voltage eliminating technique (CMVE-SVC) for three-phase multilevel voltage source inverter (MLVSI) are explored. The considered CMVE-SVC algorithm applies the particular MLVSI voltage space vectors, which produce the zero value of the common-mode voltage, i.e. the voltage across the load common point and the MLVSI common point. More specifically, the THD of the output phase voltage along with three its weighted THD of low orders are calculated in a virtual instrument of LabVIEW. The simulated dependences on the index of the amplitude modulation of the phase voltage are compared with the dependences of a recently proposed common-mode-voltage-eliminating space vector pulse width modulation technique. The importance of the considered weighted THDs for the load current THD assessment is noted.

Keywords—three-phase multilevel voltage source inverter, voltage space vector control, nearest vector selecting, common-mode voltage elimination, total harmonic distortion, integrated voltage harmonics factors, weighted total harmonic distortion

I. INTRODUCTION

Notwithstanding a great diversity of circuitry topologies of modern multilevel voltage source inverters (MLVSI), their output power quality is greatly influenced by their control algorithms. Growing requirements for the limitation of higher harmonics content of voltage and load current waveforms, the necessity of common-mode voltage (CMV) limitation and elimination, search for ways to optimize electromagnetic processes and dynamic characteristics for particular MLVSI circuits is not even close to exhaustive list of reasons, based on which a rapid development of MLVSI control techniques occurs. Starting with first MLVSI circuits' appearance and studying, many distinguished monographs, reviews and surveys on their control have been published throughout the world (see, for instance, [1–7]). Based on the analysis of the above-mentioned publications, one can easily conclude that all of the MLVSI control techniques can be roughly divided into scalar and vector ones and also into low-frequency and high-frequency ones.

The high-frequency techniques provide low values of THD and weighted THDs and make it possible to reduce a size of an output filter or even remove it from a MLVSI structure. The

low-frequency techniques reduce a number of power transistors' switchings per period and corresponding power losses in them that lets keep a high efficiency of MLVSI. One of the most widely used high-frequency methods is pulse width modulation (PWM) [2, 5, 7] in its diverse types. The low-frequency techniques include the selective harmonic elimination, the space vector control (SVC), the nearest level control, the modified nearest level modulation scheme [6] etc.

The scalar control techniques include mostly various carrier-based PWM types, which can be divided into the phase-shifted multicarrier PWM and the level-shifted multicarrier PWM, into the sinusoidal PWM and the third harmonic injected PWM [2, 3, 8]. The space vector (SV) approach deals with MLVSI space vectors and switching states in order to streamline the output voltage generating and achieve a set reference value as the average voltage value per sampling period [2, 3, 7]. The time positioning on the sampling interval of the acting of the three vectors, nearest to the reference voltage SV (RVSV), produces an additional degree of freedom that can be utilized to achieve less higher-harmonics content of the output voltage. The space vector PWM (SVPWM) compared the scalar PWM has many proven advantages, such as the better input direct voltage source usage and the increased inverter output capability. Nevertheless, there are trends towards not only the results convergence of these two techniques using but also their approaches joint applying (see, for instance, [9]).

The nearest-vector-selecting SVC (provided the number of MLVSI levels is enough high) and the low-frequency SVPWM are the two alternatives for obtaining enough low level of higher harmonics content of voltage and load current waveforms, with keeping low level of switching losses in semiconductor power switches.

II. IMPORTANCE OF THIS RESEARCH

The three above mentioned undesirable issues, namely, higher-harmonics distortion, CMV and power losses are, probably, main challenges of three-phase power generating systems based on MLVSI. The first two of them have clearly defined indicators. For example, the CMV of any three-phase MLVSI circuit is understood as the voltage across the common points of the load and of the supplying system (n and G

respectively), and it can be calculated as follows, taking into account a particular position of the node G:

$$u_{nGex}^*(t) = \frac{u_{AGex}^*(t) + u_{BGex}^*(t) + u_{CGex}^*(t)}{3}, \quad (1)$$

where u_{xG} is a phase voltage, referred to the node G; asterisks here and after indicate the normalized values, related to the unit direct voltage value U_d (base voltage level); each “exe” indicates the resulting control signal after applying the considered control technique. Its waveform must be replicated in an appropriate voltage of a power circuit.

Since the CMV is the cause of many unwelcome effects, such as leakage currents through parasitic elements, shaft voltage and bearing currents in AC motors etc. [10], its reduction and elimination in a particular MLVSI circuit is one of the targets of an applied control strategy [1-5, 7, 11-17]. The problem complete overcoming is achievable when the zero-common-mode-voltage MLVSI voltage space vectors (ZCMVSV) are being used. They correspond to $u_{nGex}^* = 0$. Though the price of such way of the elimination of the common-mode voltage is a large increase in the load voltage harmonic factors' values, increase in the number of MLVSI levels effectively mitigates this deterioration of output voltage quality.

In this situation the high harmonics level evaluation and control are coming to the fore. The conventional tool for the harmonic distortion assessment of a voltage u signal waveform is the total harmonic distortion factor (THD), which is called also the voltage harmonics factor K_{hu} and can be calculated as follows:

$$K_{hu} = \frac{U_{(hh)}}{U_{(l)}} = \frac{1}{U_{(l)}} \sqrt{\sum_{k=2}^{\infty} U_{(k)}^2} = \sqrt{\left(\frac{U}{U_{(l)}}\right)^2 - 1}, \quad (2)$$

where U , $U_{(k)}$ and $U_{(hh)}$ are the RMS values of the whole signal of estimated voltage u , its k -order harmonic component $u_{(k)}$ and higher harmonics component $u_{(hh)}$, respectively, in accordance with equation $u = u_{(l)} + u_{(hh)}$.

Unfortunately, the load current's THD, as a rule, does not correlate directly with the output voltage's THD, therefore the latter cannot be a criterion for quite sufficient quality comparative evaluation of some voltage source output voltage. The voltage THD shows only a quantity (“weight”) of higher harmonics in relation to a fundamental one and does not talk about a distribution of harmonics in a frequency range of a voltage spectrum, in particular, about their proximity to the voltage fundamental component. This is important, for instance, in some designing process, which is taking into account filtering properties of both the load itself and the specially installed circuits.

The criteria that take into account both the “weight” of high-frequency harmonics and their position in the spectrum are the integrated voltage harmonics factors (IHF), i.e. weighted THD (WTHD) factors, of various orders. They make weighted (by the harmonic order number) summation of harmonics, as a matter of fact, modeling the effect of the action of the amplitude-frequency characteristic of the idealized electric circuit of the corresponding order. Such ideal filter reduces harmonics' magnitudes by the number of times equal to the harmonic order number raised to the filter order power. That is exactly what we can see in the following equation for the n -order voltage IHF [18]:

$$\overline{K}_{hu}^{(n)} = \frac{\overline{U}_{(hh)}^{(n)}}{\overline{U}_{(l)}^{(n)}} = \frac{\overline{U}_{(hh)}^{(n)} \cdot \omega^n}{U_{(l)}} = \sqrt{\sum_{k=2}^{\infty} \left(\frac{U_{(k)}}{k^n \cdot U_{(l)}} \right)^2}, \quad (3)$$

where $\overline{U}_{(l)}^{(n)}$ and $\overline{U}_{(hh)}^{(n)}$ are the RMS values of the n times integrated MLVSI output voltage components, $u_{(l)}$ and $u_{(hh)}$ (of the fundamental circular frequency ω and of the higher harmonics, respectively).

The THD is the zero-order IHF in this system of factors, and it corresponds to a pure resistive load.

It should be noted, that Professor G.S. Zinoviev (NETI, now NSTU, Novosibirsk) has offered these IHF indices by more than 30 years ago [19], i.e. at least 10 years earlier than the well-known first order WTHD factor (the first order IHF $\overline{K}_{hu}^{(1)}$ in the considered system of factors) [2] was used for the first time (approximately in 2000).

The IHF application for the MLVSI load current's THD assessment is shown in section IV. The considered here issues have an impact on the most of the MLVSI loads functionality and efficiency, thus underscoring the importance of this research.

III. PURPOSE OF PAPER

Many of the MLVSI common-mode-voltage-eliminating control techniques apply the ZCMVSV vectors, mentioned in the previous section, see for example, [12, 16, 17].

The ZCMVSVs in coordinate frame with the new oblique-angled 60-degree axes of normalized values of MLVSI load phase-to-neutral-point voltages are presented in Fig. 1. Treating the reference vector RVSV as vector in the load phase voltages' coordinates (u_{Cn}^*, u_{An}^*) considerably simplifies both the selection of the three modulating ZCMVSV vectors and their duty-cycles in common-mode-voltage-eliminating space vector pulse width modulation (CMVE-SVPWM) [20, 21].

Provided the number of MLVSI levels is enough high, the space vector control with the nearest-vector-selecting for the common-mode voltage elimination (CMVE-SVC) can be regarded as a main competitor of the low-frequency CMVE-SVPWM.

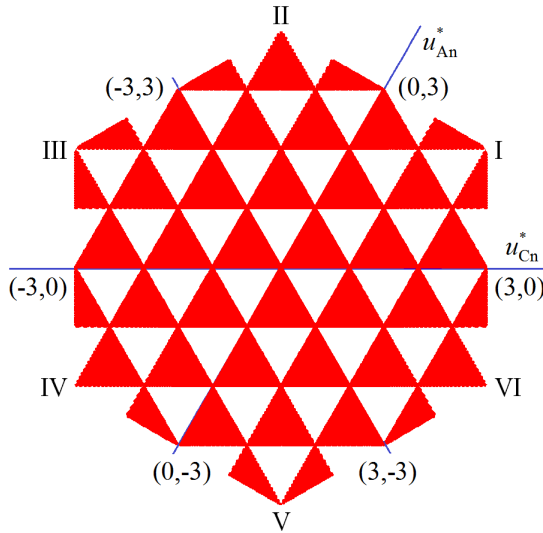


Fig. 1. Space vector diagram with triangles of the three nearest zero-common-mode-voltage space vectors of three-phase multilevel inverter in new coordinates of normalized values of load phase voltages.

The CMVE-SVC technique for MLVSI has been proposed in [12], where a significant number of calculations are needed to measure distances from RVSV position to the ZCMVSVs.

The new implementation of the CMVE-SVC makes it possible to simplify the nearest-vector-selecting calculations thanks to considering the RVSV barycentric coordinates on the triangles of the three ZCMVSV vectors nearest to the reference vector RVSV¹. Previously this approach has been proposed and studied for the case of processing and using the full set of MLVSI space vectors [22]. Regardless of the algorithmic implementation, the results of the CMVE-SVC applying are the same. But as far as we know, the only THD is considered for CMVE-SVC technique, and no IHFs' behavior investigations have already been accomplished.

The purpose of the paper is to fill this gap in the CMVE-SVC researching through the developing and using of a CMVE-SVC controller LabVIEW simulation model and the obtaining dependences of the THD and of the three weighted THDs of the lowest orders on the amplitude modulation index of the output phase voltage. These dependences will be shown in comparison with ones of the CMVE-SVPWM [21].

IV. THEORY

The IHF factors are closely related to the method of the differential equations algebraization (ADE) which allows finding asymptotically with the preset accuracy the closed analytical form expressions for the mathematical relations between the voltages and currents RMS values, using the coefficients of the integral-differential equation without solving it [18], [19].

To understand the fundamental importance of the IHF factors for the MLVSI load current's higher harmonics content assessment, we should turn to the simplified equivalent circuits of some MLVSI with its load, shown in Fig. 2.

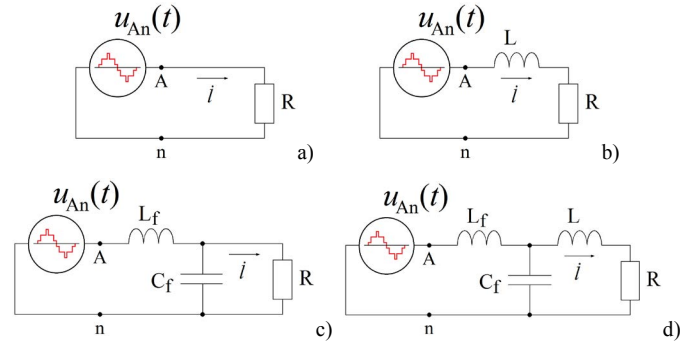


Fig. 2. Simplified per phase equivalent circuits of three-phase multilevel voltage source inverter and its loads: R-load circuit a), RL-load circuit b), R-load circuit with inverter output LC-filter c), RL-load circuit with inverter output LC-filter d).

Here $u_{An}(t)$ is the phase A MLVSI output voltage, i is the phase load current, L and R are the phase load inductance and resistance, L_f and C_f are the elements of phase MLVSI output LC-filter.

After applying the ADE procedure to differential equations, corresponding to the circuits of Fig. 2, first approximation estimates of the MLVSI phase load current's THD (the factor of current's harmonics K_{hi}) have been derived and here summarized in Table I. The first approximation means deletion of terms with once integrated higher harmonics component $i_{(hh)}$ in a resulting integral equation in accordance with the "filter hypothesis" [18]. The similar THD assessing can be performed for any element of any equivalent circuit regarding the current in this element or the voltage across it.

As can be seen, each estimate of load current's THD is directly proportional to the exposure voltage's IHF of the order (from zero to three), equal to the order of a differential equation corresponding to the considered equivalent circuit of a generalized load. Thus, the set of the lower-order IHF factors of the generated MLVSI voltage provides its description, necessary for the load power quality assessment and prediction in a particular MLVSI-based AC power supply system.

TABLE I. EQUATIONS FOR ASSESSMENT OF LOAD CURRENT THD

Equivalent Circuit of MLVSI With Its Load	Factor of Load Current's Harmonics K_{hi} (THD)
R-load circuit	$\bar{K}_{hu}^{(0)} = K_{hu}$
RL-load circuit	$\bar{K}_{hu}^{(1)} \cdot \left[1 + (R/(\omega L))^2 \right]^{1/2}$
R-load circuit with inverter output LC-filter	$\bar{K}_{hu}^{(2)} \cdot \left[1 + \left(\frac{1}{(RC_f)^2} - \frac{2}{L_f C_f} \right) \frac{1}{\omega^2} + \frac{1}{(L_f C_f)^2 \omega^4} \right]^{1/2}$
RL-load circuit with inverter output LC-filter	$\bar{K}_{hu}^{(3)} \cdot \left[1 + \frac{a_1^2 - 2a_2}{\omega^2} + \frac{a_2^2 - 2a_1 a_3}{\omega^4} + \frac{a_3^2}{\omega^6} \right]^{1/2}$, $a_1 = \frac{R}{L}$, $a_2 = \frac{1}{C_f} \left(\frac{1}{L_f} + \frac{1}{L} \right)$, $a_3 = \frac{R}{L_f C_f L}$

1. N. N. Lopatkin, "New implementation of common-mode-voltage-eliminating nearest-vector-selecting space vector control for three-phase multilevel inverter," 2021 Int. Ural Conf. on Electrical Power Engineering, 2021, in press.

It's surprising, but IHF factors still haven't become a conventional tool, widely used in related standards and in engineering practice.

V. SIMULATION RESULTS

The CMVE-SVC controller simulation model is here implemented in LabVIEW, since numerous LabVIEW simulations and developed virtual IHF-meter have proved to be useful and convenient². The controller model, represented in Fig. 3, has mostly replicated the respective Matlab/Simulink model¹. Here $b_{Xn} = \{u_{Xn}^*\}$, i.e. the fractional part of u_{Xn}^* ; m_{aY} is the phase voltage amplitude modulation index,

$$m_{aY} = U/U_d = U^*, \quad (4)$$

U and U^* are the value and relative value of the reference vector RVSV magnitude; the signals denoted by u_{XGex}^* are the respective executed signals for phase legs to be replicated as power circuit phase-to-neutral voltages u_{Xn}^* (here $u_{Xn}^* = u_{XG}^*$).

The new implementation of the CMVE-SVC technique¹ uses the two-component principle of the executed voltages shaping. For some sampling moment each of the two output phase-to-ground voltages, treated here as the executed voltage space vector coordinates, is calculated as follows:

$$u_{XGex}^* = u_{Xnexe}^* = \lfloor u_{Xnref}^* \rfloor + f_{Xnexe}, \quad (5)$$

where u_{Xnref}^* is the sampled reference voltage signal; $\lfloor u_{Xnref}^* \rfloor$ is its integer part; f_{Xnexe} functions are defined as follows:

$$\begin{aligned} f_{Cnexe} &= (F_{up} \cap f_{Cn1} \cap f_{Cn2}) \cup (\bar{F}_{up} \cap (f_{Cn1} \cup f_{Cn2})) \\ f_{Anexe} &= (F_{up} \cap f_{An} \cap \bar{f}_{Cn1}) \cup (\bar{F}_{up} \cap (f_{An} \cup \bar{f}_{Cn1})) \\ f_{Cn1} &\equiv \{u_{Cn}^*\} > \{u_{An}^*\}, \\ f_{Cn2} &\equiv \{u_{Cn}^*\} > \{u_{Bn}^*\}, \\ f_{An} &\equiv \{u_{An}^*\} > \{u_{Bn}^*\} \end{aligned} \quad (6)$$

F_{up} is the upward-pointing NTV triangle type identifier, $\bar{w}$ is the logical inversion of w .

The used in the developed model analog control might be treated as digital one with the tending to zero sampling period duration. For more detailed information on the above presented approach, please refer to [22] and to the references in it.

The CMVE-SVC simulated IHFs' dependences of the u_{An}^* waveform on the amplitude modulation index m_{aY} are presented in Fig. 4 and Fig. 5 up to the $m_{aY \max} = 10$. Thus, the curves provide the data on the content of the higher harmonics for the output voltage of a 21-level MLVS circuit. The curves of the same dependences of the CMVE-SVPWM [21] for the three values of the frequency modulation index m_f are placed in the figures for comparisons. Here $m_f = f_c/f$, where f_c and f are the clock-cycle and modulating frequencies, respectively.

It can be seen that the considered CMVE-SVC is a quite competitive technique. It has a clear advantage in the voltage THD values, therefore it should be recommended for use in MLVSI circuits with nearly pure-resistive generalized loads. Regarding the rest of the factors, the CMVE-SVC is inferior to the CMVE-SVPWM having the frequency modulation index values, that beginning at least from 24 for the first m_{aY} subrange and from 36 for the second one. This is due to the fact that the pulses of a more high frequency can be filtered out with a greater efficiency.

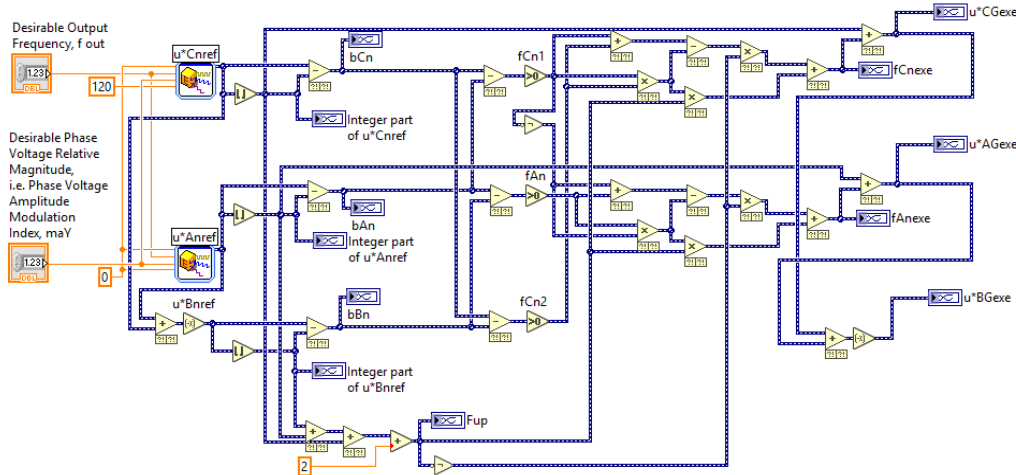


Fig. 3. LabVIEW common-mode-voltage-eliminating nearest voltage space vector selecting controller for three-phase multilevel voltage source inverter with arbitrary level number.

2. N. N. Lopatkin, I. S. Lucenko, and Yu. A. Chernov, "Virtual instrument for assessment of simulated signal integrated harmonics factors," XIV Int. Sci.-Tech. Conf. on Actual Problems of Electronics Instrument Engineering, pp. 152–157, 2018.

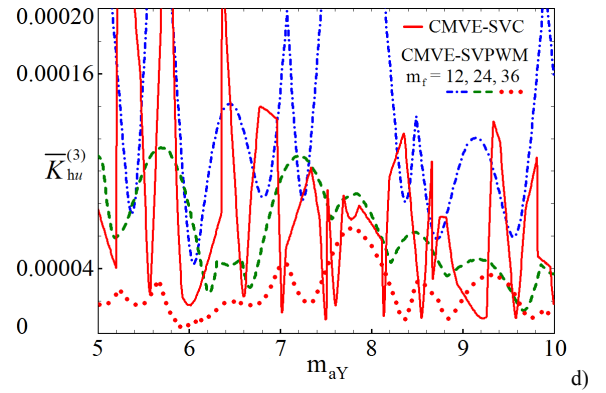
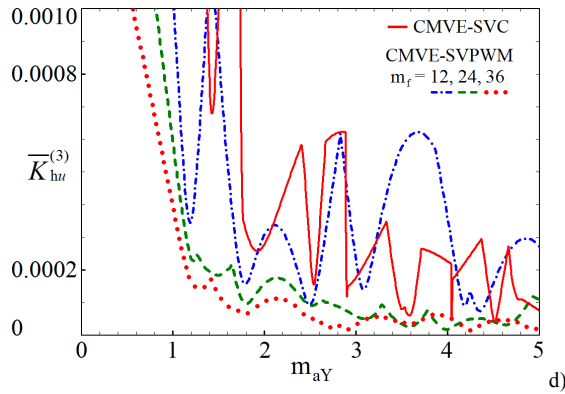
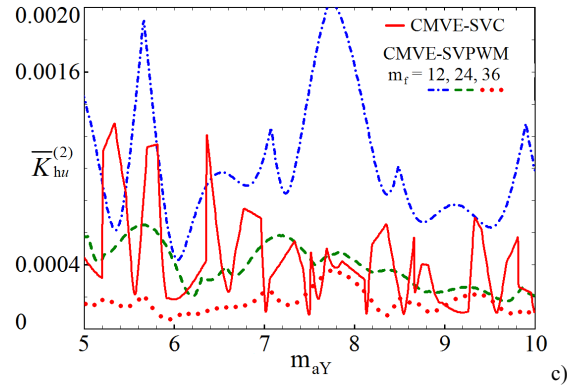
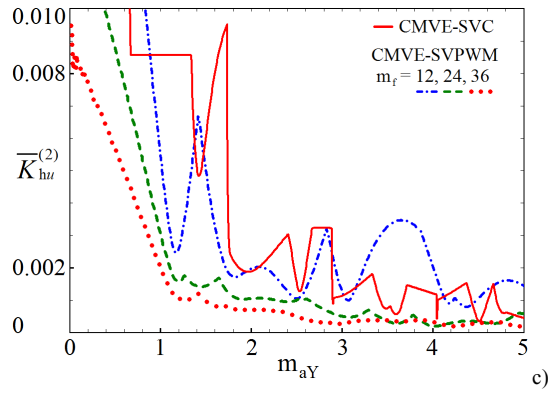
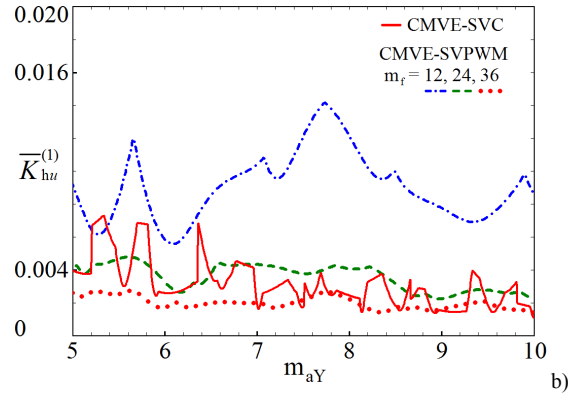
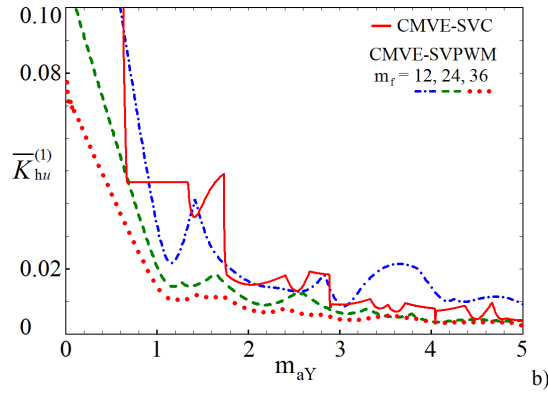
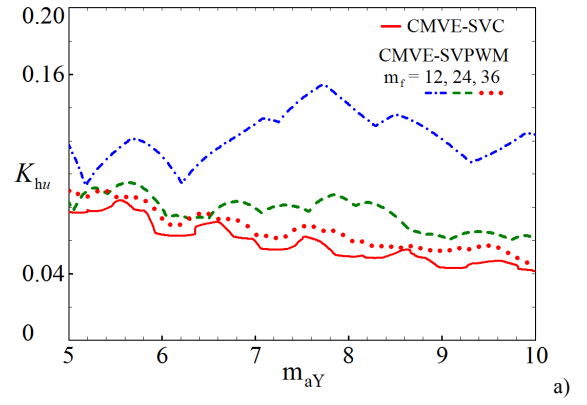
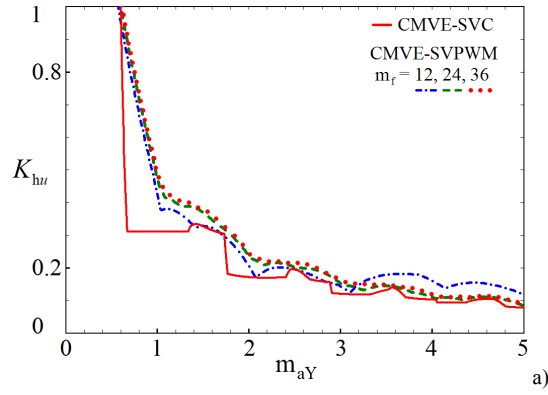


Fig. 4. CMVE-SVC and CMVE-SVPWM simulated dependences on amplitude modulation index in its subrange 0...5: THD (a), IHF from the first to the third orders (b, c, and d).

Fig. 5. CMVE-SVC and CMVE-SVPWM simulated dependences on amplitude modulation index in its subrange 5...10: THD (a), IHF from the first to the third orders (b, c, and d).

VI. CONCLUSION

The nearest-vector-selecting space vector control, which uses the only particular voltage space vectors with the zero value of the common-mode voltage (i.e. the voltage across the load common point and the MLVSI common point), is considered. The relationships between amplitude modulation index of the phase voltage and its THD and integrated voltage harmonics factors values (weighted THDs) have been obtained by simulations in LabVIEW. They have been carried out for the initial amplitude range, namely, up to the line-mode range limit of a 21-level voltage source inverter. The dependences make it possible to evaluate beforehand the THD value of the current in the inverter load circuit's element or of the voltage across it.

It is now clear that this space vector control technique with common-mode voltage eliminating has advantage in the voltage THD values over the recently proposed space vector pulse width modulation, which uses the same set of the space vectors. Thus, it can be applied for any case where a reactance of a general equivalent circuit (of an inverter and its load) may be neglected. Regarding the load circuits, having filtering properties, deterioration of their load current waveform's quality is balanced against the less switching power losses. So, development engineer of the particular power system would be able to set the relevant priorities. With an increase in the number of inverter levels, the voltages generated by the two methods under consideration tend to approaches one another in the integrated voltage harmonics factors' values. Therefore, the issues of the power losses and efficiency should come to the fore.

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