

Assessment of Conductive EMI of Nonlinear Consumers of Common DC Bus with PWM

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Abstract—The assessment method of the DC-circuit analogues for the voltage and current THD values is here generalized to an arbitrary number of nonlinear loads. This technique is based on the method of the differential equations algebraization, and it makes it possible not only to predict the quality of power supply at the design stage for the power supply system with known analytical descriptions of nonlinear consumers' currents, but also to assess their individual relative share contributions in the overall distortion of power quality. The analytical expression for the DC-circuit arbitrary order integrated signal harmonics factor's dependence on the amplitude modulation index is obtained for the PWM-modulated pulse signal by using the infinite series harmonic synthesis formulae. The lowest orders' DC-circuit harmonic factors' dependences on the amplitude modulation index are presented and verified through PSIM computer simulations. The ripples of the supply voltage at the filter output generated by two common DC bus nonlinear loads in the form of PWM-modulated current sources at varied phase shifts of the PWM carrier triangle signals are estimated for the PSIM model particular parameters.

Keywords—DC-circuit THD analogue, DC-circuit n -order integrated signal harmonics factor, common DC bus, nonlinear consumers, pulse-width power DC-DC converter, PWM-modulated pulse signal, individual load relative share contribution to AC component, method of differential equations algebraization

I. INTRODUCTION

The conductive electromagnetic interference (EMI) problems becomes actual for DC circuits in cases where mono-bus DC system, i.e. a common DC bus, feeds multiple nonlinear consumers [1], especially when the itself DC bus voltage control is performed with use of PWM. The conducted EMI noise is assessed by means of some coefficients related to the signal harmonic content.

Reference [2] has offered the method for calculating electromagnetic compatibility indicators of nonlinear consumers of a common DC bus with PWM which are the DC-circuit analogues for the voltage and current THD values.

The employed approach is based on the method of the differential equations algebraization (ADE) [3]. The general ADE technique allows obtaining a closed analytical form asymptotic expressions (with a needed accuracy) for the RMS voltages and currents values without solving the differential equations itself. To use the ADE method the researcher needs some generic data on harmonic content of the circuit exposure signals, namely of the signals of voltage and current sources, presenting the circuit supply and loads. A family of weighted total harmonic distortion coefficients, so called differential and integral factors of harmonics (the differentiated and integrated harmonics factors), acts as the

required exposure description. These factors make it possible for us to simulate the action of ideal differentiating and integrating circuits of various orders. The new integrated voltage and current harmonics factors, intended to DC circuits, are presented in [2].

The method suggested in [2] is also providing the assess of the nonlinear loads individual relative share contributions in the overall harmonic distortion of DC power quality.

The purpose of this paper is to generalize the equations for DC-circuit harmonic indicators, related to the ripple voltages and ripple currents, and for the loads individual relative share contributions for the case of arbitrary number N of nonlinear loads. Also the objectives are to consider the DC-circuit various order integrated harmonics factors for a PWM-modulated pulse exposure signal, and to present some simulation result regarding the DC voltage quality at various phase shift values of the PWM carrier triangle signals. In addition to the above mentioned generalization, the research scientific novelty is related to deriving the expression for the DC-circuit arbitrary order integrated harmonics factor of the symmetric PWM-modulated pulse signal.

II. INTEGRATED HARMONICS FACTORS FOR PWM-MODULATED PULSE SIGNAL

The various order integrated signal harmonics factors reflect the load filtering effect on the investigated signal. They makes weighted (by the harmonic order number) summation of harmonics, as a matter of fact, modelling the effect of the action of the amplitude-frequency characteristic of the idealized electric circuit of the corresponding order. Such the ideal filter reduces the magnitudes of every harmonic by the number of times equal to the harmonic order number raised to the filter order power.

For some exposure variable x (specified value) and some arbitrary order n the DC-circuit integrated signal harmonics factor $\overline{K}_{\text{hxDC}}^{(n)}$ is defined as follows:

$$\overline{K}_{\text{hxDC}}^{(n)} = \frac{\overline{X}_{\text{AC}}^{(n)} \cdot \omega_x^n}{X_{\text{DC}}} = \frac{1}{X_{\text{DC}}} \sqrt{\sum_{k=1}^{\infty} \left(\frac{X_{(k)}}{k^n} \right)^2}, \quad (1)$$

where $\overline{X}_{\text{AC}}^{(n)}$ is the RMS value of n times integrated AC component of an exposure x , ω_x is the circular frequency of its fundamental harmonic, $X_{(k)}$ is the RMS value of k -order harmonic of x and X_{DC} is the average value (the DC component) of x .

The DC-circuit n -order integrated harmonics coefficient $\overline{K}_{\text{hxDC}}^{(n)}$ can be also expressed by means of the long been known AC-circuit n -order coefficient $\overline{K}_{\text{hx}}^{(n)}$ [3]:

$$\overline{K}_{\text{hxDC}}^{(n)} = \frac{X_{(1)}}{X_{\text{DC}}} \cdot \sqrt{1 + \left(\overline{K}_{\text{hx}}^{(n)}\right)^2}. \quad (2)$$

We now turn to study a PWM-modulated pulse exposure signal. Consider, as an example of the PWM signal, the symmetric signal shown in Fig. 1. Here, in accordance with the well-known property of any even signal [4], one can derive the expression for the instantaneous PWM-modulated signal value in the form of an infinite series:

$$x(\vartheta) = X_0 \left(M + \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{\sin M\pi k \cdot \cos k\vartheta}{k} \right), \quad (3)$$

where X_0 is the value of the initial constant level of a physical quantity which carries the signal (voltage or current), M is the amplitude modulation index (the amplitude modulation depth), $M = S_{\text{ref}}/S_{\text{pp}} = X_{\text{DC}}/X_0$, S_{ref} and S_{pp} are the reference level value and the peak-to-peak amplitude of the triangle signal s_{triangle} , respectively.

Substitution the RMS value of k -order harmonic of x in (1) by the RMS value of the k -th term of series (3) produces the following equation, again in the form of an infinite series:

$$\overline{K}_{\text{hxDC}}^{(n)} = \frac{\sqrt{2}}{M\pi} \sqrt{\sum_{k=1}^{\infty} \frac{\sin^2 M\pi k}{k^{2(n+1)}}}. \quad (4)$$

The analytical description (the expression in the form of the series which contains only few terms) for the DC-circuit arbitrary order integrated harmonics factor's dependence on the amplitude modulation index is obtained for the considered PWM-modulated pulse signal by using the infinite series harmonic synthesis formulae [5]:

$$\overline{K}_{\text{hxDC}}^{(n)}(M) = \sqrt{2}(2\pi)^n \frac{\sqrt{A(M)}}{M},$$

$$A(M) = \frac{|B_{2(n+1)}|}{(2(n+1))!} - (-1)^n \sum_{p=0}^{2(n+1)} \frac{B_p M^{2(n+1)-p}}{p!(2(n+1)-p)!}, \quad (5)$$

where the first Bernoulli numbers are $B_0 = 1$, $B_1 = -1/2$, $B_2 = 1/6$, $B_3 = B_5 = B_7 = B_9 = 0$, $B_4 = B_8 = -1/30$, $B_6 = 1/42$, $B_{10} = 5/66$.

The equation (5) is valid under the condition $0 < M < 1$, i.e. for the full range of values.

The curves of the zero to fourth orders DC-circuit integrated signal harmonics factors dependences on the amplitude modulation depth M are shown in Fig. 2. The zero-order index $\overline{K}_{\text{hxDC}}^{(0)}$ is, as a fact, the DC-circuit THD analogue, but here it is related to the exposure x itself.

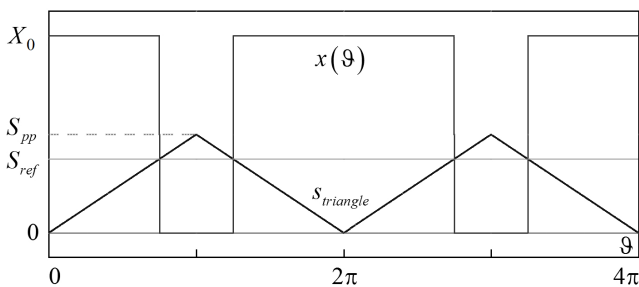


Fig. 1. Symmetric PWM signal pulses generation.

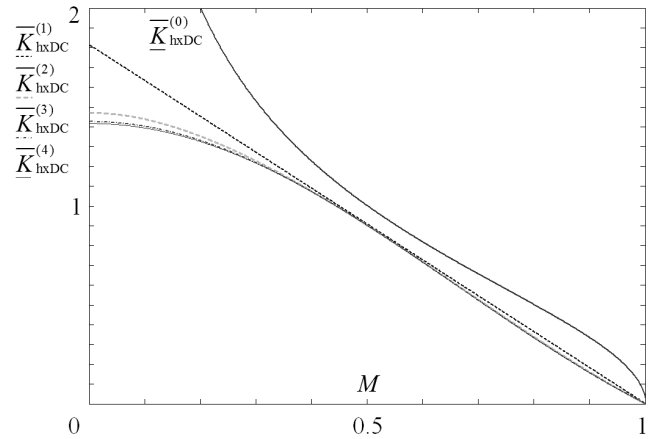


Fig. 2. DC-circuit integrated signal harmonics factors dependences on the amplitude modulation depth for symmetric PWM signal.

The first-order index $\overline{K}_{\text{hxDC}}^{(1)}$ function is a linear one:

$$\overline{K}_{\text{hxDC}}^{(1)}(M) = \frac{\pi}{\sqrt{3}}(1-M). \quad (6)$$

The rest of the indices, except for the zero-order one, blend into the general curve which is enough close to the straight line of the index $\overline{K}_{\text{hxDC}}^{(1)}$, beginning approximately from the M value of 0.45.

The above presented dependences' formulae and curves may be useful to industrial engineers who design some power supply system with a common bus, based on the pulse-width power converter with output LC -filter and use the calculation technique discussed in the next section.

III. PULSE-WIDTH POWER DC-DC CONVERTER WITH OUTPUT LC -FILTER

The equivalent circuit of a pulse-width power converter with output LC -filter loaded by nonlinear consumers is shown in Fig. 3.

The differential equations, establishing the relationships between the instantaneous values of the voltage u across the capacitor C (of the bus voltage), of the current i in the filter reactor L , of the PWM power supply EMF e and of the nonlinear consumers' currents i_1, i_2, \dots, i_N are:

$$L \frac{di}{dt} + u = e, \quad i = C \frac{du}{dt} + i_1 + i_2 + \dots + i_N. \quad (7)$$

Using designations, similar to applied in (1), we can write the equations for the DC components of the required variables u and i at once:

$$U_{\text{DC}} = E_{\text{DC}}, \quad I_{\text{DC}} = I_{1\text{DC}} + I_{2\text{DC}} + \dots + I_{N\text{DC}}. \quad (8)$$

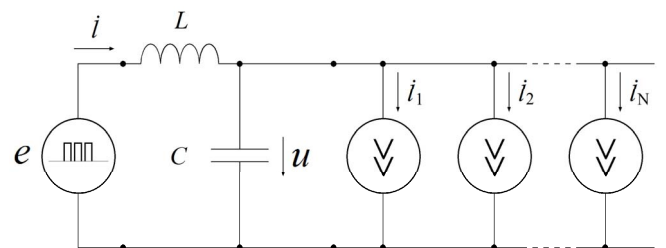


Fig. 3. Equivalent circuit of power supply system with common bus of device circuit board, based on pulse-width power DC-DC converter.

Considering (7) for AC components of voltages e_{AC} , u_{AC} and currents i_{1AC} , i_{2AC} , ..., i_{NAC} , i_{AC} lead to deducing the following equations:

$$u_{AC} + \frac{1}{LC} u_{AC}^{-(2)} = \frac{1}{LC} e_{AC}^{-(2)} - \frac{1}{C} \left(i_{1AC}^{-(1)} + i_{2AC}^{-(1)} + \dots + i_{NAC}^{-(1)} \right), \quad (9)$$

$$i_{AC} + \frac{1}{LC} i_{AC}^{-(2)} = \frac{1}{L} e_{AC}^{-(1)} + \frac{1}{LC} \left(i_{1AC}^{-(2)} + i_{2AC}^{-(2)} + \dots + i_{NAC}^{-(2)} \right), \quad (10)$$

where $y_{AC}^{-(n)}$ is the result of the n -fold indefinite integral taking of the instantaneous AC variable y_{AC} .

In the context of the ADE technique [3], after rejection of the terms with integrated AC components of required variables (the first, i.e. the roughest level of approximation is adopted), applying the squaring and averaging operators and using (1) we obtain algebraic equations regarding to the AC components RMS values of voltages and currents, i.e. ripple voltages and ripple currents:

$$U_{AC}^2 = \sum_{p=1}^N \left(\frac{\overline{K}_{hipDC}^{(1)}}{\omega_p C} \cdot I_{pDC} \right)^2 + \frac{2}{C^2} \sum_{\substack{j,k=1 \\ k>j}}^N \left(i_{jAC}^{-(1)} \cdot i_{kAC}^{-(1)} \right) + \left(\frac{\overline{K}_{heDC}^{(2)}}{\omega^2 LC} \cdot E_{DC} \right)^2 - \frac{2}{LC^2} \sum_{s=1}^N \left(i_{sAC}^{-(1)} \cdot e_{AC}^{-(2)} \right), \quad (11)$$

$$I_{AC}^2 = \sum_{p=1}^N \left(\frac{\overline{K}_{hipDC}^{(2)}}{\omega_p^2 LC} \cdot I_{pDC} \right)^2 + \frac{2}{(LC)^2} \sum_{\substack{j,k=1 \\ k>j}}^N \left(i_{jAC}^{-(2)} \cdot i_{kAC}^{-(2)} \right) + \left(\frac{\overline{K}_{heDC}^{(1)}}{\omega L} \cdot E_{DC} \right)^2 + \frac{2}{L^2 C} \sum_{s=1}^N \left(i_{sAC}^{-(2)} \cdot e_{AC}^{-(1)} \right), \quad (12)$$

where N is an arbitrary number of nonlinear consumers; ω and $\omega_1, \omega_2, \dots, \omega_N$ are the circular frequencies of the fundamental harmonics of the change of the PWM-modulated voltage e and of the nonlinear consumers currents, respectively; the expressions in parentheses are the scalar products of the corresponding exposure variables:

$$\left(x_j^{-(m)}, x_k^{-(n)} \right) = \frac{1}{T_0} \int_0^{T_0} x_j^{-(m)} x_k^{-(n)} dt, \quad (13)$$

T_0 is an averaging period.

The considered here conductive electromagnetic compatibility indicators for the DC power supply with nonlinear load are the DC-circuit voltage and current THD analogues:

$$K_{huDC} = \frac{U_{AC}}{U_{DC}}, \quad K_{hiDC} = \frac{I_{AC}}{I_{DC}}, \quad (14)$$

where the average values of the required variables are defined in (8).

The coefficients K_{huDC} and K_{hiDC} can be treated as zero-order integrated harmonics factors or general ripple factors, that, in contrast to the conventional ripple factor, take into

account not only the harmonics with the highest signal value but also all the rest harmonics.

The obtained analytical equations for the electromagnetic compatibility indicators allow to predict the quality of power supply at the design stage for the power supply system with known analytical descriptions of any form currents and of any arbitrary quantity of nonlinear consumers.

Moreover, we can deduce the coefficients of participation of individual nonlinear consumers in the overall distortion of power quality in the power supply system, based on (11) and (12).

The contributions of the exposure x (the PWM-modulated voltage e and the nonlinear consumer i_k , $k \in \{1, 2, \dots, N\}$) to the square of the bus voltage AC component V_x and to the square of the power supply current AC component J_x are, respectively,

$$V_e = V_e^e + \sum_{k=1}^N V_e^{i_k}, \quad V_{i_k} = \sum_{s=1}^N V_{i_k}^{i_s} + V_{i_k}^e = V_{i_k}^{i_k} + \sum_{\substack{s=1 \\ s \neq k}}^N V_{i_k}^{i_s} + V_{i_k}^e, \quad (15)$$

$$J_e = J_e^e + \sum_{k=1}^N J_e^{i_k}, \quad J_{i_k} = \sum_{s=1}^N J_{i_k}^{i_s} + J_{i_k}^e = J_{i_k}^{i_k} + \sum_{\substack{s=1 \\ s \neq k}}^N J_{i_k}^{i_s} + J_{i_k}^e, \quad (16)$$

where V_x^x and J_x^x are the own contributions of the exposure x to U_{AC}^2 and to I_{AC}^2 ,

$V_{x_1}^{x_2}$ and $J_{x_1}^{x_2}$ are the shares of the joint contributions of the exposures x_1 and x_2 to U_{AC}^2 and to I_{AC}^2 in V_{x_1} and in J_{x_1} ,

$$V_{x_1}^{x_2} = V_{x_1, x_2} \frac{V_{x_1}^{x_1}}{V_{x_1}^{x_1} + V_{x_2}^{x_2}}, \quad J_{x_1}^{x_2} = J_{x_1, x_2} \frac{J_{x_1}^{x_1}}{J_{x_1}^{x_1} + J_{x_2}^{x_2}}, \quad (17)$$

V_{x_1, x_2} and J_{x_1, x_2} are the above mentioned joint contributions of the exposures x_1 and x_2 to U_{AC}^2 and to I_{AC}^2 . Obviously, that $V_{x_1}^{x_2} + V_{x_2}^{x_1} = V_{x_1, x_2}$, $J_{x_1}^{x_2} + J_{x_2}^{x_1} = J_{x_1, x_2}$.

In particular, as we can see in (11),

$$V_{i_k}^{i_k} = \left(\frac{\overline{K}_{hikDC}^{(1)}}{\omega_k C} \cdot I_{kDC} \right)^2, \quad V_{i_k, i_s} = \frac{2}{C^2} \left(i_{kAC}^{-(1)} \cdot i_{sAC}^{-(1)} \right),$$

$$V_e^e = \left(\frac{\overline{K}_{heDC}^{(2)}}{\omega^2 LC} \cdot E_{DC} \right)^2, \quad V_{e, i_k} = -\frac{2}{LC^2} \left(i_{kAC}^{-(1)} \cdot e_{AC}^{-(2)} \right),$$

$$V_e^{i_k} = V_{e, i_k} \frac{V_e^e}{V_e^e + V_{i_k}^{i_k}}, \quad V_{i_k}^{i_s} = V_{i_k, i_s} \frac{V_{i_k}^{i_k}}{V_{i_k}^{i_k} + V_{i_s}^{i_s}}, \quad V_{i_k}^e = V_{e, i_k} \frac{V_{i_k}^{i_k}}{V_e^e + V_{i_k}^{i_k}}.$$

Similarly, in accordance with (12),

$$J_{i_k}^{i_k} = \left(\frac{\overline{K}_{hikDC}^{(2)}}{\omega_k^2 LC} \cdot I_{kDC} \right)^2, \quad J_{i_k, i_s} = \frac{2}{(LC)^2} \left(i_{kAC}^{-(2)} \cdot i_{sAC}^{-(2)} \right),$$

$$J_e^e = \left(\frac{\overline{K}_{heDC}^{(1)}}{\omega L} \cdot E_{DC} \right)^2, \quad J_{e, i_k} = \frac{2}{L^2 C} \left(i_{kAC}^{-(2)} \cdot e_{AC}^{-(1)} \right),$$

$$J_{e,i_k}^{i_k} = J_{e,i_k} \frac{J_e^e}{J_e^e + J_{i_k}^{i_k}}, \quad J_{i_k,i_s}^{i_s} = J_{i_k,i_s} \frac{J_{i_k}^{i_k}}{J_{i_k}^{i_k} + J_{i_s}^{i_s}}, \quad J_{i_k}^e = J_{e,i_k} \frac{J_{i_k}^{i_k}}{J_e^e + J_{i_k}^{i_k}}.$$

The corresponding relative share contributions (the individual coefficients of participation) to the overall DC voltage and current distortions (ripples) can be defined as follows:

$$K_u(x) = \frac{V_x}{U_{AC}^2}, \quad K_i(x) = \frac{J_x}{I_{AC}^2}. \quad (18)$$

Obviously, for the relative share contributions of all the exposures e and i_1, i_2, \dots, i_N the following equations are true:

$$K_u(e) + \sum_{k=1}^N K_u(i_k) = 1, \quad K_i(e) + \sum_{k=1}^N K_i(i_k) = 1. \quad (19)$$

The example of the calculation of the relative share contributions to the overall DC voltage and current distortions for the case of the only two nonlinear DC power consumers is presented in [2].

IV. PSIM SIMULATION FOR TWO PWM-MODULATED CURRENT SOURCES OF LOADS.

A computer model, intended to assess values of the THD analogue K_{huDC} of the supply DC voltage at the filter output, is shown in Fig. 4.

The voltage ripples, except for the ones that are produced by DC-DC converter itself, are here generated by two nonlinear loads in the form of PWM-modulated current sources, that can change a value of the phase shift between the PWM carrier triangle signals of the same fundamental frequency. To find a main regularity the same values of the amplitude modulation index of the current sources have been set.

The ripples voltage RMS value U_{AC} can be obtained by means of much more simple way than in (11), i.e.

$$U_{AC} = \sqrt{U^2 - U_{DC}^2}, \quad (20)$$

where U is the RMS value of the whole signal of the voltage u across the capacitor C .

As such, here the conductive electromagnetic compatibility indicator for the DC power supply, namely the DC-circuit voltage THD analogue, has been calculated as follows:

$$K_{huDC} = \sqrt{\left(\frac{U}{U_{DC}}\right)^2 - 1}. \quad (21)$$

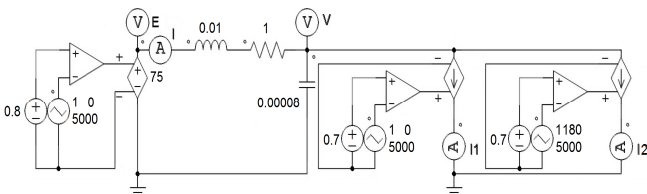


Fig. 4. PSIM computer model of PWM-modulated power DC-DC converter loaded by two nonlinear consumers in the form of PWM-modulated current sources.

Unfortunately, this approach excludes a finding values of the share contributions of the PWM-modulated voltage and the nonlinear consumers to the AC component of a bus voltage.

The simulation results are presented in Fig. 5 and Fig. 6. Here α is a phase delay between the triangle carrier signals of the first and second PWM-modulated loads' current sources, β is a phase delay between the triangle carrier signals of the first PWM-modulated load current source and of the PWM power supply EMF e .

It should be noted, that the results relates mostly to this particular circuit example and to the PSIM model particular parameters, which, however, can be enough close to real circuits. Whereas the EMF e ripples generate the ripples of the consumed current i , the pulse width modulated currents i_1 and i_2 are making the dominant contribution to the decline in voltage u quality. In effect, their phase shift of PWM carrier triangle signals provide the level of the voltage THD analogue K_{huDC} . The phase delay α value, corresponding to the local minimums of K_{huDC} , is situated near π radians and becomes biased due to the phase delay β changing. The lower K_{huDC} values than reached in Fig. 5 value of 0.001478, namely about 0.001454, are demonstrated in Fig. 6 for the case of the fixed level $\alpha=\pi$ and the fitted β values.

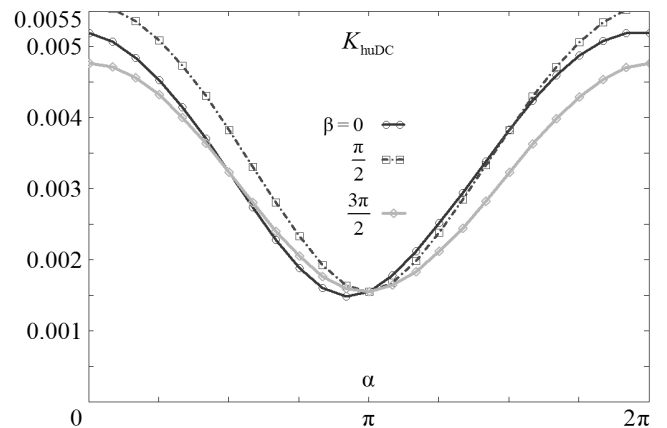


Fig. 5. Dependences of DC-circuit voltage THD analogue on phase shift of PWM carrier triangle signals of PWM-modulated load current sources.

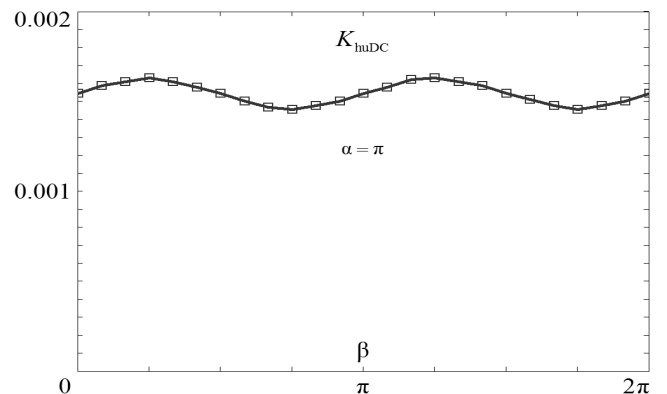


Fig. 6. Dependence of DC-circuit voltage THD analogue on phase shift of PWM carrier triangle signals of PWM-modulated load current source and of PWM power supply EMF.

Of course, a proportion of the amplitude modulation index values of the current sources strongly affects the value of the voltage THD analogue K_{huDC} . To provide a more definitive conclusions about ways of search of some global minimum of K_{huDC} , we should carry out some more systematic computer experiments and apply the analysis with use of the mathematical instruments of (5) and (15).

V. CONCLUSIONS

The assessment method of the DC-circuit THD-analogues for voltage and current of a mono-bus DC system, including the equations for DC-circuit harmonic indicators, related to the ripple voltages and ripple currents, and for the loads individual relative share contributions, is generalized to an arbitrary number of nonlinear loads.

The analytical expression for the DC-circuit arbitrary order integrated signal harmonics factor's dependence on the amplitude modulation index for the PWM-modulated pulse signal in case of the triangle carrier signal is obtained. The equation has been verified through PSIM computer simulations.

The ripples of the supply voltage at the filter output generated by two common DC bus nonlinear loads in the form of PWM-modulated current sources at various phase shifts of the PWM carrier triangle signals are estimated for

the PSIM model particular parameters. It is shown that the optimization of the DC power quality by varying parameters of the carrier signals of PWM-modulated nonlinear consumers is promising.

The suggested dependences' formulae and curves may be useful for industrial engineers, who design some power supply system with a common bus, based on the pulse-width power converter with output LC-filter, and with known analytical descriptions of multiple of harmonic-producing loads, due to opportunities to predict and to improve the quality of power supply at the design stage.

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