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# Applying barycentric coordinates on balanced supply voltages triangle to zero sequence voltage estimate without phase measurements

**N N Lopatkin**

Department of Mathematics, Physics and Informatics, Shukshin Altai State University for Humanities and Pedagogy, 53, Korolenko str., Biysk, 659333, Russia

E-mail: nikolay\_lopatkin@mail.ru

**Abstract.** The paper offers a new vector-coordinate description for a complex-plane position of a neutral bias voltage phasor relevant to a three-phase circuit with an unbalanced load. It is assumed that no negative sequence voltage component is present, i.e. the supply three-phase system voltages are symmetric and sinusoidal. The so called “new coordinates” and being defined with the help of them barycentric coordinates on a balanced supply line-to-line voltages triangle have been introduced. The proposed approach made it possible to obtain simple enough analytical expressions. A RMS value of the positive sequence component and further a static phasor of the zero sequence voltage unbalance factor are described. They are based on three line-to-neutral load voltages voltmeter RMS readings only. Accordingly, these calculations need no phase measurements. The presented neutral bias voltage coordinates calculations might be useful in a three-phase load circuit monitoring, diagnostics (fault detection), protection and voltages unbalance compensation.

## 1. Introduction

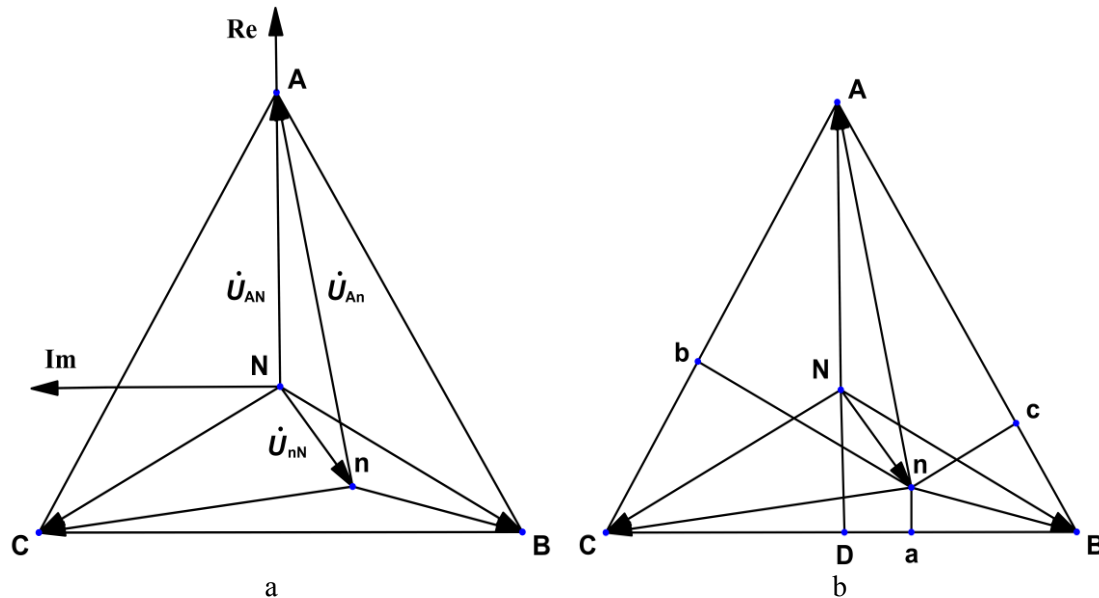
An unbalance related to voltages and currents is one of the most significant aspects of the three-phase power quality. It affects not only electrical equipment operability and performances but also components safety and lifetime. A presence of a neutral bias voltage is just one of the voltage unbalance kinds. It occurs in three-phase three-wire and four-wire circuits with an unbalanced load and is characterized by a zero sequence voltage component. Despite the fact this unbalance kind is widespread mode, the European regulations are mostly concentrate on a negative sequence voltage component definition and detection regarding line-to-line voltages. For example, one can refer to the International Electrotechnical Commission standard IEC 60034-26.

This paper proposes an innovative vector-coordinate description for a complex-plane position of a neutral bias voltage phasor. The case with the absence of any negative sequence voltage component, i.e. when supply three-phase system voltages are symmetric and sinusoidal, is studied. As a result, a practical engineering tool to fulfil a three-phase load circuit monitoring, diagnostics (fault detection), protection and voltages unbalance compensation is provided. The so called “voltmeter approach”, i.e. measurements with no phase data use, is applied here. At first, a static phasor of a zero sequence component of the load phase voltages has been defined. Then the redefined RMS value of the positive sequence component is obtained. At last, a static phasor of the zero sequence voltage unbalance factor has been described thoroughly.



## 2. Barycentric coordinates on balanced supply voltages triangle

A position typical for an unbalanced load of a load neutral or common point  $n$  phasor is shown in phasor diagrams in figure 1. It is associated with a complex plane and with a basic (reference) triangle of balanced supply line-to-line voltages. Here the vertexes A, B and C are phasors ends of balanced phase voltages, i.e. supply line-to-neutral voltages, related to a supply neutral or common point N.



**Figure 1.** Voltage phasors of a three-phase circuit with symmetric supply and unbalanced load: a – representation in a complex plane; b – representation regarding barycentric coordinates on a triangle.

Hereinafter  $\dot{XY}$  designates the complex-plane phasor directed from X to Y and related to the corresponding RMS voltage static phasor  $\dot{U}_{YX}$  (figure 1,a):

$$\dot{XY} = \dot{U}_{YX} = \dot{\phi}_Y - \dot{\phi}_X. \quad (1)$$

Here  $\dot{\phi}_X$  and  $\dot{\phi}_Y$  are complex-plane phasors of X and Y nodes electric potentials, relatively, and  $XY$  designates the  $\dot{XY}$  absolute value and the corresponding voltage RMS value  $U_{YX}$ .

It seems fruitful to use mapping of coordinates on a basic triangle (figure 1,b) onto the complex plane phasor diagram. In particular, since N is the centroid (geometric center, barycenter) of the  $\Delta ABC$ , a well-known equation can be applied:

$$nA^2 + nB^2 + nC^2 = NA^2 + NB^2 + NC^2 + 3 \cdot Nn^2. \quad (2)$$

Accordingly, a RMS value of a neutral bias voltage can be expressed:

$$U_{An}^2 + U_{Bn}^2 + U_{Cn}^2 = 3(U_{ph}^2 + U_{nN}^2), \quad U_{nN} = \left( \frac{U_{An}^2 + U_{Bn}^2 + U_{Cn}^2}{3} - U_{ph}^2 \right)^{1/2}. \quad (3)$$

Here  $U_{An}$ ,  $U_{Bn}$  and  $U_{Cn}$  are measured load line-to-neutral (phase) voltages RMS values;  $U_{ph}$  is here supposed to be a rated (nominal) supply line-to-neutral voltage RMS value,  $U_{AN} = U_{BN} = U_{CN} = U_{ph}$ .

If an area of  $\Delta ABC$  is assumed to be equal to 1, weighting coefficients, equal to areas of  $\Delta AnB$ ,  $\Delta BnC$  and  $\Delta CnA$ , are areal coordinates or barycentric coordinates ( $u_n, v_n, w_n$ ) of point n on  $\Delta ABC$ , which can be defined as follows:

$$u_n = h_A/H = na/H, \quad v_n = h_B/H = nb/H, \quad w_n = h_C/H = nc/H, \quad (4)$$

where  $H$  is the altitude's length (height) of the equilateral triangle,  $H = AD \equiv \frac{3}{2} \cdot U_{ph}$ .

Wherever the point  $n$  is located, its barycentric coordinates fulfill the condition:

$$u_n + v_n + w_n = 1. \quad (5)$$

While the point  $n$  is situated inside the  $\triangle ABC$ , values of all its barycentric coordinates  $(u_n, v_n, w_n)$  remain within the interval from 0 to 1. Otherwise, one or two of the coordinates become negative together with one or two of respective heights  $h_A$ ,  $h_B$  and  $h_C$ .

The barycentric coordinates  $(u_n, v_n, w_n)$  could serve as a convenient tool for voltage unbalance monitoring and control. In particular, if one of them is equal to 1, it means a short circuit of a corresponding load phase. The zero value indicates the load phase open circuit.

The disappointing drawback of the mathematic formalization of the barycentric coordinates  $(u_n, v_n, w_n)$ , as in equations (4), is assignment of the sign to every of heights. It implies an apriory point  $n$  position knowledge.

Fortunately, with the help of the so called "new coordinates", a new definition of the barycentric coordinates on a triangle has been suggested in the plane geometry textbook [1]. And in a later-dated issued scholarly book [2] of the certain author, Stanislav Petrovich Shkroba, the same is discussed. The "new coordinates"  $x$ ,  $y$  and  $z$  are precisely related to the load phase voltages RMS values:

$$x = nC^2 - nB^2 = U_{Cn}^2 - U_{Bn}^2, \quad y = nA^2 - nC^2 = U_{An}^2 - U_{Cn}^2, \quad z = nB^2 - nA^2 = U_{Bn}^2 - U_{An}^2, \quad x + y + z = 0 \quad (6)$$

The results for the barycentric coordinates of the point  $n$  on the equilateral triangle are:

$$u_n = \frac{1}{3} \left( 1 + \frac{U_{Bn}^2 + U_{Cn}^2 - 2U_{An}^2}{3U_{ph}^2} \right), \quad v_n = \frac{1}{3} \left( 1 + \frac{U_{Cn}^2 + U_{An}^2 - 2U_{Bn}^2}{3U_{ph}^2} \right), \quad w_n = \frac{1}{3} \left( 1 + \frac{U_{An}^2 + U_{Bn}^2 - 2U_{Cn}^2}{3U_{ph}^2} \right). \quad (7)$$

The following equivalent equations demonstrate the coordinate decomposition of the neutral bias voltage phasor by the basis  $(\dot{U}_{AN}, \dot{U}_{BN}, \dot{U}_{CN})$ :

$$N\dot{n} = u_n \dot{N}A + v_n \dot{N}B + w_n \dot{N}C, \quad \dot{U}_{nN} = u_n \dot{U}_{AN} + v_n \dot{U}_{BN} + w_n \dot{U}_{CN}. \quad (8)$$

Accordingly, the load phase voltages RMS phasors can be presented in the same non-orthogonal basis expansion:

$$\dot{U}_{An} = \dot{U}_{AN} + \dot{U}_{Nn} = \dot{U}_{AN} - \dot{U}_{nN} = (1 - u_n) \dot{U}_{AN} - v_n \dot{U}_{BN} - w_n \dot{U}_{CN}, \quad (9)$$

$$\dot{U}_{Bn} = \dot{U}_{BN} + \dot{U}_{Nn} = \dot{U}_{BN} - \dot{U}_{nN} = -u_n \dot{U}_{AN} + (1 - v_n) \dot{U}_{BN} - w_n \dot{U}_{CN}, \quad (10)$$

$$\dot{U}_{Cn} = \dot{U}_{CN} + \dot{U}_{Nn} = \dot{U}_{CN} - \dot{U}_{nN} = -u_n \dot{U}_{AN} - v_n \dot{U}_{BN} + (1 - w_n) \dot{U}_{CN}. \quad (11)$$

To distinguish by sign rises above from falls below the central point  $N$  coordinates  $(1/3, 1/3, 1/3)$  in point  $n$  coordinates  $(u_n, v_n, w_n)$  deviations, shifted barycentric coordinates  $(u'_n, v'_n, w'_n)$  are suited:

$$u'_n = u_n - 1/3, \quad v'_n = v_n - 1/3, \quad w'_n = w_n - 1/3, \quad u'_n + v'_n + w'_n = 0. \quad (12)$$

Such variables, taking into account both values and polarities of the deviation coordinates, can be used as signals for compensator to take appropriate action.

The number of observed variables of point  $n$  can be reduced to two due to equation (5) and the use of a pair of the barycentric coordinates, e.g.  $u_n$  and  $v_n$ , as the affine (oblique-angled) coordinates:

$$C\dot{n} = v_n C\dot{B} + u_n C\dot{A} = v_n \dot{U}_{BC} - u_n \dot{U}_{CA} = \sqrt{3}U_{ph}(v_n e^{-i\frac{\pi}{2}} + u_n e^{-i\frac{\pi}{6}}) = \frac{3}{2}U_{ph}[u_n - i\frac{1}{\sqrt{3}}(u_n + 2v_n)]. \quad (13)$$

### 3. Zero and positive sequence voltage components

After denoting  $e^{i\frac{2\pi}{3}} = \dot{a}$  the basis voltage phasors can be represented as follows:

$$\dot{U}_{AN} = U_{ph}, \quad \dot{U}_{BN} = \dot{a}^2 U_{ph}, \quad \dot{U}_{CN} = \dot{a} U_{ph}, \quad (14)$$

so the equations (8)-(11) can be written as:

$$\dot{U}_{nN} = U_{ph}(u_n + v_n \dot{a}^2 + w_n \dot{a}), \quad (15)$$

$$\dot{U}_{An} = U_{ph}[(1 - u_n) - v_n \dot{a}^2 - w_n \dot{a}], \quad (16)$$

$$\dot{U}_{Bn} = U_{ph}[-u_n + (1 - v_n) \dot{a}^2 - w_n \dot{a}], \quad (17)$$

$$\dot{U}_{Cn} = U_{ph}[-u_n - v_n \dot{a}^2 + (1 - w_n) \dot{a}]. \quad (18)$$

The positive  $\dot{U}_1$ , negative  $\dot{U}_2$  and zero  $\dot{U}_0$  symmetric components of the load phase voltages RMS phasors can be obtained in accordance with the Fortescue transform [3]:

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \\ \dot{U}_0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \dot{a} & \dot{a}^2 \\ 1 & \dot{a}^2 & \dot{a} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{U}_{An} \\ \dot{U}_{Bn} \\ \dot{U}_{Cn} \end{bmatrix}. \quad (19)$$

The results of the equations (16)-(18) substitution to equation (19) are quite expected both for the positive and negative sequence components, namely,  $\dot{U}_1 = U_{ph} = \dot{U}_{AN}$ ,  $\dot{U}_2 = 0$ . But rather new information can be seen for the zero sequence component, while taking into account equations (7):

$$\dot{U}_0 = \dot{U}_{Nn} = -\dot{U}_{nN} = -U_{ph}(u_n + v_n \dot{a}^2 + w_n \dot{a}) = \frac{1}{3U_{ph}} \left[ U_{An}^2 - \frac{1}{2}(U_{Bn}^2 + U_{Cn}^2) - i\frac{\sqrt{3}}{2}(U_{Bn}^2 - U_{Cn}^2) \right]. \quad (20)$$

The zero sequence phasor polar form is defined by the expression:

$$\dot{U}_0 = U_0 e^{i\alpha}, \quad (21)$$

where the modulus  $U_0$  and the argument  $\alpha$  may be derived from equation (20).

In particular, the expression for the zero sequence voltage RMS, equal to the neutral bias voltage RMS value, takes the form:

$$U_0 = |\dot{U}_0| = U_{Nn} = U_{nN} = \frac{1}{3U_{ph}} \left[ U_{An}^4 + U_{Bn}^4 + U_{Cn}^4 - (U_{An}^2 U_{Bn}^2 + U_{Bn}^2 U_{Cn}^2 + U_{Cn}^2 U_{An}^2) \right]^{1/2}. \quad (22)$$

Thus, we have equations (3) and (22), which are providing calculation of the same value by the different ways. The high agreement degree of the calculations results has been checked by Mathcad model and it points to some hidden geometric relationship.

The zero sequence phasor can be also represented directly in "new coordinates":

$$\dot{U}_0 = \frac{1}{6U_{ph}}(y-z+i\sqrt{3}x), U_0 = \frac{1}{6U_{ph}}[(y-z)^2+3x^2]^{1/2}, \tan \alpha = \frac{\sqrt{3}x}{y-z} = \frac{\sqrt{3}(U_{Cn}^2-U_{Bn}^2)}{2U_{An}^2-U_{Bn}^2-U_{Cn}^2}. \quad (23)$$

Equation (3) is much simpler than equation (22) and suitable for the zero sequence voltage RMS calculation. Despite it, the using of the beforehand known rated supply phase voltage RMS value  $U_{ph}$  affects the accuracy of the assessment when supply voltages change without a symmetry lose. But now we are able to obtain the exact expression for the phase voltage RMS value after the simultaneous use of equations (3) and (22). Provided that the condition  $U_0 \leq U_{ph}$  is fulfilled for the values, based on the rated supply phase voltage RMS value  $U_{ph}$ , the refined values can be calculated as follows:

$$U_1 = \dot{U}_1 = (A+B)^{1/2}/\sqrt{2}, U_0 = (A-B)^{1/2}/\sqrt{2}. \quad (24)$$

$$\text{Here } A = \frac{U_{An}^2 + U_{Bn}^2 + U_{Cn}^2}{3}, B = \left[ \frac{1}{3}(2C-D) \right]^{1/2}, C = U_{An}^2 U_{Bn}^2 + U_{Bn}^2 U_{Cn}^2 + U_{Cn}^2 U_{An}^2,$$

$$D = U_{An}^4 + U_{Bn}^4 + U_{Cn}^4.$$

#### 4. Static phasor of the zero sequence voltage unbalance factor

Instead of conventional consideration of unbalance factors magnitudes, a wide range of specialists in electrical engineering started using corresponding phasors. It makes possible to control both generated and consumed power. So, using equations (21), (23) and (20), here we introduce the static phasor of the zero sequence component voltage unbalance factor:

$$\dot{K}_{0U} = \frac{\dot{U}_0}{\dot{U}_1} = \frac{U_0}{U_{ph}} e^{i\alpha} = K_{0U} e^{i\alpha}, \quad (25)$$

$$\dot{K}_{0U} = \frac{1}{6U_{ph}^2}(y-z+i\sqrt{3}x) = -(u_n + v_n \dot{a}^2 + w_n \dot{a}), \quad (26)$$

$$\dot{K}_{0U} = \frac{1}{3U_{ph}^2} \left[ U_{An}^2 - \frac{1}{2}(U_{Bn}^2 + U_{Cn}^2) - i \frac{\sqrt{3}}{2}(U_{Bn}^2 - U_{Cn}^2) \right]. \quad (27)$$

Assuming the phase voltage RMS value  $U_{ph}$  as the base value for voltages and their phasors, i.e.:

$$U_x^* = \frac{U_x}{U_{ph}}, \dot{U}_x^* = \frac{\dot{U}_x}{U_{ph}}, \quad (28)$$

we can readily rewrite the last expression in the following form:

$$\dot{K}_{0U} = \dot{U}_0^* = \frac{1}{3} \left\{ (U_{An}^*)^2 - \frac{1}{2} [(U_{Bn}^*)^2 + (U_{Cn}^*)^2] - i \frac{\sqrt{3}}{2} [(U_{Bn}^*)^2 - (U_{Cn}^*)^2] \right\}. \quad (29)$$

The argument of  $\dot{K}_{0U}$  is the same value  $\alpha$ , and it corresponds to equation (23). The modulus as the conventional zero sequence component voltage unbalance factor now can be described in the following ways:

$$K_{0U} = \frac{U_0}{U_{ph}} = \frac{1}{6U_{ph}^2} [(y-z)^2 + 3x^2]^{1/2} = (u_n^2 + v_n^2 + w_n^2 - u_n v_n - v_n w_n - w_n u_n)^{1/2}, \quad (30)$$

$$K_{0U} = \frac{1}{3U_{ph}^2} (U_{An}^4 + U_{Bn}^4 + U_{Cn}^4 - U_{An}^2 U_{Bn}^2 - U_{Bn}^2 U_{Cn}^2 - U_{Cn}^2 U_{An}^2)^{1/2}, \quad (31)$$

$$K_{0U} = \frac{1}{3} [(U_{An}^*)^4 + (U_{Bn}^*)^4 + (U_{Cn}^*)^4 - (U_{An}^* U_{Bn}^*)^2 - (U_{Bn}^* U_{Cn}^*)^2 - (U_{Cn}^* U_{An}^*)^2]^{1/2}. \quad (32)$$

The more compact form is derived from equation (3):

$$K_{0U} = \frac{U_{nN}}{U_{ph}} = \left( \frac{U_{An}^2 + U_{Bn}^2 + U_{Cn}^2}{3U_{ph}^2} - 1 \right)^{1/2} = \left[ \frac{(U_{An}^*)^2 + (U_{Bn}^*)^2 + (U_{Cn}^*)^2}{3} - 1 \right]^{1/2}. \quad (33)$$

At last, equations (24) with the redefined sequence components for the case of the supply voltages changes without a symmetry lose, provide the respective expression:

$$K_{0U} = \left( \frac{A-B}{A+B} \right)^{1/2}. \quad (34)$$

In addition, both equations (30) could be reduced to a correspondent two-coordinate presentation. Furthermore, the barycentric form passes from the three-coordinate to the two-coordinate symmetry of the equation:

$$K_{0U} = [3(u_n^2 + v_n^2 + u_n v_n - u_n - v_n) + 1]^{1/2}. \quad (35)$$

And obviously, the argument of  $\dot{K}_{0U}$  also can be represented with help of the barycentric coordinates of the point n:

$$\tan \alpha = \frac{\sqrt{3}(w_n - v_n)}{2u_n - v_n - w_n} = \frac{\sqrt{3}(w_n - v_n)}{3u_n - 1} = \frac{\sqrt{3}(1 - u_n - 2v_n)}{3u_n - 1}. \quad (36)$$

Suppose, for example, that the line-to-neutral load voltages RMS voltmeter readings are

$$U_{An} = 199.81 \text{ V}, U_{Bn} = 234.38 \text{ V}, U_{Cn} = 217.77 \text{ V}.$$

Applying equations (24) and (34) produces values, which are rounded here to the nearest two decimal places (the accuracy of the initial data):

$$U_1 = U_{ph} = 216.86 \text{ V}, U_0 = 19.98 \text{ V}, K_{0U} = 0.09 \text{ or } 9\%.$$

Since  $U_{ph}$  has been defined, any of the above described phasors  $\dot{U}_0$  and  $\dot{K}_{0U}$  coordinate representations become accessible for use.

## 5. Conclusion

The complex plane coordinates binding to positive sequence component voltage phasor has been implemented for the phasors of the neutral bias voltage, the load phase voltages and the zero sequence voltage unbalance factor. The multiple coordinate's representations are provided for an appropriate voltage control, which is related to generated and consumed power. The high efficiency of this approach has been proved, in the case of the absence of the negative sequence voltage component. The voltages RMS values of the positive and zero sequences have been found from the readings of the three load phase voltages RMS values. The further development of this coordinate description seems to be useful to calculation and control of other unsymmetrical modes of three-phase circuits.

## References

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