Zero Sequence Voltage Phasor Positioning under Balanced Three-Phase Supply

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***Abstract*—Based on the representations in barycentric coordinates on a balanced supply line-to-line voltages triangle the resulting complex-plane static phasors of the zero sequence voltage and of the related unbalance factor are obtained for the balanced mode of the three-phase supplying. It is supposed that the voltages and currents are sinusoidal. The suggested formulae calculations require only RMS voltmeter readings of the three line-to-neutral load voltages and could be used as a practical engineering tool to implement a three-phase load circuit monitoring, diagnostics, protection and voltages unbalance compensation. Physical interpretation of the neutral bias voltage phasor barycentric coordinates, corresponding to the coefficients of participation of the individual phase loads in this bias voltage generating, is also presented as the functions of the phase loads electrical admittances parameters.**

Keywords*—*unbalanced load, neutral bias voltage, zero sequence voltage, RMS values, voltage zero sequence unbalance factor, barycentric coordinates, static phasor of the voltage zero sequence unbalance factor

I. Introduction

Specific for polyphase power supply systems and critical for the efficient and safe operation of three-phase power consumers is the aspect of the electromagnetic compatibility problem related to the unbalance of voltages and currents of the three-phase system. The corresponding European regulations [1]–[3] and one way or another related to three-phase unbalance numerous comparatively recent publications both here and abroad (for example, refer to [4]–[10]) confirm the complexity of the tasks in this area and the urgent need to solve them.

While approaches to the definition and reducing of the unbalance in the presence of higher harmonics are outlined and are being implemented [11]–[14], the most simple kind of three-phase unbalance, namely, the case of a neutral bias voltage due to unbalanced three-phase load in a sinusoidal mode, seems to be described unsufficiently. In that respect, the expressions, analytically describing the zero sequence voltage phasor and somewhat similar to presented in [5] for negative sequence voltage phasor, would be useful to have. This paper proposes such the complex-plane description for positioning of the neutral bias voltage phasor under the balanced sinusoidal three-phase supplying voltages. The offered coordinates calculations of the load common node potential are intended to perform monitoring, diagnostics (fault detection), protection and load voltages unbalance compensation, in particular by means of power electronics devices.

II. Neutral Bias Voltage

Fig. 1 shows the simplified equivalent circuit of the four-wire three-phase star-connected circuit, which, as is well known, is valid under the following conditions:

- there is only a static load (no electric motors);
- voltage drops in generators are not taken into account.

Here and throughout $\dot{z}_\mathrm{A}$, $\dot{z}_\mathrm{B}$, $\dot{z}_\mathrm{C}$ and $\dot{z}_\mathrm{N}$ are the load impedances of the phases A, B, C and the neutral wire impedance, respectively, $\dot{z}_\mathrm{A} = R_\mathrm{A} + jX_\mathrm{A} = Z_\mathrm{A}\mathrm{e}^{j\cdot\varphi_\mathrm{A}}$ etc.; $\dot{U}_\mathrm{AN}$, $\dot{U}_\mathrm{BN}$, $\dot{U}_\mathrm{CN}$ are the phasors of the phases A, B, C supplying voltages (which are referred to the three-phase source common star point N); $\dot{U}_\mathrm{An}$, $\dot{U}_\mathrm{Bn}$, $\dot{U}_\mathrm{Cn}$ are the phasors of the phases A, B, C load voltages (which are referred to the three-phase load common star point n); $\dot{U}_\mathrm{nN}$ is the neutral bias voltage phasor, which can be calculated as follows [15]:

$$\dot{U}_\mathrm{nN} = \frac{\dot{y}_\mathrm{A}\dot{U}_\mathrm{AN} + \dot{y}_\mathrm{B}\dot{U}_\mathrm{BN} + \dot{y}_\mathrm{C}\dot{U}_\mathrm{CN}}{\dot{y}_\mathrm{A} + \dot{y}_\mathrm{B} + \dot{y}_\mathrm{C} + \dot{y}_\mathrm{N}}, \quad (1)$$

where $\dot{y}_\mathrm{A}$, $\dot{y}_\mathrm{B}$, $\dot{y}_\mathrm{C}$ and $\dot{y}_\mathrm{N}$ are the admittances of the phases A, B, C loads and the neutral wire, respectively, $\dot{y}_\mathrm{A} = 1/\dot{z}_\mathrm{A}$ etc.

If the neutral wire is absent ($\dot{y}_\mathrm{N} = 0$), then the circuit becomes the three-wire one.

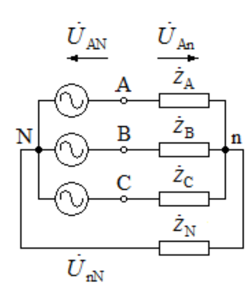

Fig. 1. Equivalent circuit of four-wire three-phase star-connected circuit.

The neutral bias voltage occurs in unbalanced both the four-wire and the three-wire three-phase star-connected circuits. The phasor diagram for some unbalanced three-phase star-connected circuit is shown in Fig. 2.

III. Barycentric Coordinates on Balanced Supply Line-to-Line Voltages Triangle

Hereinafter the supplying three-phase voltages, both line-to-line and line-to-neutral, are assumed balanced, and the load voltages unbalance is just due to the load impedances and admittances unbalance.

The phasors of Fig. 2 can be considered from the geometrical point of view, namely, as related to the regular triangle of the line-to-line voltages and being in the context of the barycentric coordinates on this triangle, see Fig. 3.

This approach can be fruitful for the search of the new relations between the vectors (phasors) lengths, which are corresponding to the voltages RMS values:

$$XY = \left|\dot{XY}\right| = \left|\dot{U}_{\mathrm{YX}}\right| = U_{\mathrm{YX}} .$$

If an area of ΔABC is assumed to be equal to 1, weighting coefficients, equal to areas of ΔAnB, ΔBnC and ΔCnA, are areal coordinates or barycentric coordinates (un,vn,wn) of point n on ΔABC, which can be defined as follows:

$$\begin{aligned} u_{\mathrm{n}} &= h_{\mathrm{A}}/H = na/H \\ v_{\mathrm{n}} &= h_{\mathrm{B}}/H = nb/H \\ w_{\mathrm{n}} &= h_{\mathrm{C}}/H = nc/H \end{aligned} , \quad (2)$$

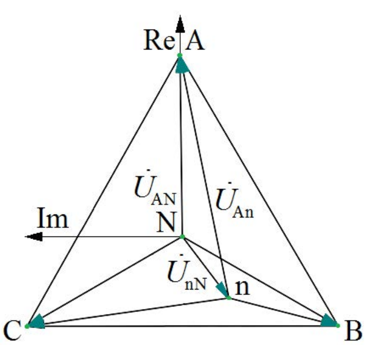

Fig. 2. Phasors of voltages in the context of the complex plane.

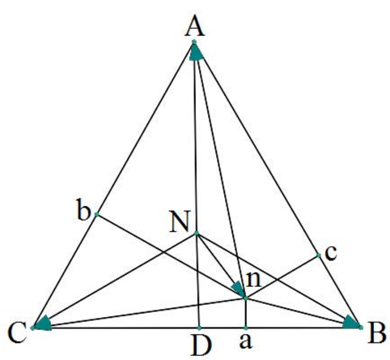

Fig. 3. Phasors of voltages in the context of the reference triangle.

where H is the height of ΔABC, $H = AD = \sqrt{3}/2 \cdot U_{\mathrm{ph}}$, U_{ph} is the RMS value of the phases A, B, C balanced supplying voltages, which are referred to the common point N.

Wherever the point n is located, its barycentric coordinates fulfill the condition:

$$u_{\mathrm{n}} + v_{\mathrm{n}} + w_{\mathrm{n}} = 1 . \quad (3)$$

While the point n is situated inside the ΔABC, values of all its barycentric coordinates (u_{n},v_{n},w_{n}) remain within the interval from 0 to 1. Otherwise, one or two of the coordinates become negative together with one or two of respective heights h_{A}, h_{B} and h_{C}.

The barycentric coordinates (u_{n},v_{n},w_{n}) could serve as a convenient tool for voltage unbalance monitoring and control. In particular, if one of them is equal to 1, it means a short circuit of a corresponding load phase. The zero value indicates the load phase open circuit.

The disappointing drawback of the mathematic formalization of the barycentric coordinates (u_{n},v_{n},w_{n}), as in equations (2), is assignment of the sign to every of the heights. It implies an apriory point n position knowledge.

Recently issued textbook [16] and scholarly book on the coordinate geometry in a plane [17] have presented so-called “new coordinates” and a new definition of the barycentric coordinates on a triangle.

The “new coordinates” x, y and z are the quantities composed of the squared values, which are the absolute values of the phasors of the phases A, B, C load voltages (referred to the three-phase load common star point n):

$$\begin{aligned} x &= nC^2 - nB^2 = U_{\mathrm{Cn}}^2 - U_{\mathrm{Bn}}^2 \\ y &= nA^2 - nC^2 = U_{\mathrm{An}}^2 - U_{\mathrm{Cn}}^2 \\ z &= nB^2 - nA^2 = U_{\mathrm{Bn}}^2 - U_{\mathrm{An}}^2 \end{aligned} , \quad (4)$$

The condition for the “new coordinates”, similar to (3), is

$$x + y + z = 0 . \quad (5)$$

The barycentric coordinates on an arbitrary line-to-line voltages triangle are expressed through the “new coordinates” [16], [17] and can be written in terms of voltage RMS values, but the equations form has turned out to be quite complex. Here see the equation for the first coordinate as the example:

$$\begin{aligned} & u_{\mathrm{n}} = Num/Denom , \\ & Num = (U_{\mathrm{AB}}^2 - U_{\mathrm{CA}}^2)(U_{\mathrm{Cn}}^2 - U_{\mathrm{Bn}}^2) + \\ & + U_{\mathrm{BC}}^2 (U_{\mathrm{CA}}^2 + U_{\mathrm{AB}}^2 - U_{\mathrm{BC}}^2 + U_{\mathrm{Bn}}^2 + U_{\mathrm{Cn}}^2 - 2U_{\mathrm{An}}^2), \\ & Denom = 2(U_{\mathrm{AB}}^2 U_{\mathrm{BC}}^2 + U_{\mathrm{BC}}^2 U_{\mathrm{CA}}^2 + U_{\mathrm{CA}}^2 U_{\mathrm{AB}}^2) - \\ & -(U_{\mathrm{AB}}^4 + U_{\mathrm{BC}}^4 + U_{\mathrm{CA}}^4) \end{aligned} , \quad (6)$$

Here the line-to-line voltages RMS values U_{AB}, U_{BC} and U_{CA} are needed for calculations.

Since for the equilateral reference triangle

$$U_{AB} = U_{BC} = U_{CA} = \sqrt{3} \cdot U_{ph} \quad (7)$$

the barycentric coordinates on balanced supply line-to-line voltages triangle are determined in much more simple form:

$$\begin{aligned} u_n &= \frac{1}{3}\left(1 + \frac{U_{Bn}^2 + U_{Cn}^2 - 2U_{An}^2}{3U_{ph}^2}\right) \\ v_n &= \frac{1}{3}\left(1 + \frac{U_{Cn}^2 + U_{An}^2 - 2U_{Bn}^2}{3U_{ph}^2}\right), \\ w_n &= \frac{1}{3}\left(1 + \frac{U_{An}^2 + U_{Bn}^2 - 2U_{Cn}^2}{3U_{ph}^2}\right) \end{aligned} \quad (8)$$

The following equivalent equations demonstrate the coordinate decomposition of the neutral bias voltage phasor by the basis $(\dot{U}_{AN}, \dot{U}_{BN}, \dot{U}_{CN})$:

$$\begin{aligned} N\dot{n} &= u_n N\dot{A} + v_n N\dot{B} + w_n N\dot{C} \\ \dot{U}_{nN} &= u_n \dot{U}_{AN} + v_n \dot{U}_{BN} + w_n \dot{U}_{CN} \end{aligned}, \quad (9)$$

It should be noted, however, that this barycentric representation of the point n position requires the knowledge not only of the load voltages values but also of the value of the phase-to-ground balanced supplying voltages.

IV. Load Voltages Symmetrical Components

The positive $\dot{U}_1$, negative $\dot{U}_2$ and zero $\dot{U}_0$ symmetrical components of the load phase voltages RMS phasors can be obtained in accordance with the Fortescue transform [18]:

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \\ \dot{U}_0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \dot{a} & \dot{a}^2 \\ 1 & \dot{a}^2 & \dot{a} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{U}_{An} \\ \dot{U}_{Bn} \\ \dot{U}_{Cn} \end{bmatrix}, \quad (10)$$

where $\dot{a} = e^{j \cdot \frac{2\pi}{3}}$.

In accordance with Fig. 1 and (9),

$$\begin{aligned} \dot{U}_{An} &= \dot{U}_{AN} - \dot{U}_{nN} = (1-u_n)\dot{U}_{AN} - v_n \dot{U}_{BN} - w_n \dot{U}_{CN} \\ \dot{U}_{Bn} &= \dot{U}_{BN} - \dot{U}_{nN} = -u_n \dot{U}_{AN} + (1-v_n)\dot{U}_{BN} - w_n \dot{U}_{CN} \\ \dot{U}_{Cn} &= \dot{U}_{CN} - \dot{U}_{nN} = -u_n \dot{U}_{AN} - v_n \dot{U}_{BN} + (1-w_n)\dot{U}_{CN} \end{aligned} \quad (11)$$

After substitutions of the basis voltage phasors expression

$$\begin{bmatrix} \dot{U}_{AN} \\ \dot{U}_{BN} \\ \dot{U}_{CN} \end{bmatrix} = U_{ph} \begin{bmatrix} 1 \\ \dot{a}^2 \\ \dot{a} \end{bmatrix} \quad (12)$$

into (11) and then (11) into (10), the result for the symmetrical components is as follows:

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \\ \dot{U}_0 \end{bmatrix} = \begin{bmatrix} U_{ph} \\ 0 \\ -\dot{U}_{nN} \end{bmatrix}. \quad (13)$$

So, the barycentric representation and, according to (8), the complex-plane representation for the phasor of the load voltages zero sequence component are obtained:

$$\begin{aligned} \dot{U}_0 &= \dot{U}_{Nn} = -\dot{U}_{nN} = -U_{ph}(u_n + v_n \dot{a}^2 + w_n \dot{a}) = \\ &= \frac{1}{3U_{ph}}\left[U_{An}^2 - \frac{1}{2}(U_{Bn}^2 + U_{Cn}^2) - j\frac{\sqrt{3}}{2}(U_{Bn}^2 - U_{Cn}^2)\right] \end{aligned} \quad (14)$$

And for the zero sequence voltage RMS value we have

$$\begin{aligned} U_0 &= \left|\dot{U}_0\right| = U_{Nn} = U_{nN} = \\ &= \frac{\sqrt{U_{An}^4 + U_{Bn}^4 + U_{Cn}^4 - (U_{An}^2 U_{Bn}^2 + U_{Bn}^2 U_{Cn}^2 + U_{Cn}^2 U_{An}^2)}}{3U_{ph}} \end{aligned} \quad (15)$$

Generally, the exact RMS value of the phase-to-ground balanced supplying voltages U_{ph} is unknown, therefore we should write the additional equation, which can be obtained through the well-known geometrical relation for Fig. 3. Namely, since N is the centroid (geometric center, barycenter) of the ΔABC, and given that $U_{AN} = U_{BN} = U_{CN} = U_{ph}$, one may conclude that

$$\begin{aligned} nA^2 + nB^2 + nC^2 &= NA^2 + NB^2 + NC^2 + 3 \cdot Nn^2, \\ U_{An}^2 + U_{Bn}^2 + U_{Cn}^2 &= 3(U_{ph}^2 + U_{nN}^2) \end{aligned} \quad (16)$$

and finally we have

$$U_0 = U_{nN} = \sqrt{\frac{U_{An}^2 + U_{Bn}^2 + U_{Cn}^2}{3} - U_{ph}^2}. \quad (17)$$

The same can be obtained for the supply balanced mode from one of the formulae in [5].

The Table I summarizes the more accurate expressions, which have been found by solving the system of equations (15) and (16) for the values of $U_{ph} = U_1$ and U_0.

TABLE I. POSITIVE AND ZERO SYMMETRICAL COMPONENTS

Condition and Mode	Symmetrical Components RMS Values	
	U_1	U_0
$A < 2U_{ph\_r}^2$, $U_0 < U_1$	$\sqrt{\frac{A+B}{2}}$	$\sqrt{\frac{A-B}{2}}$
$A = 2U_{ph\_r}^2$, $U_0 = U_1$	$\sqrt{\frac{A}{2}}$	$\sqrt{\frac{A}{2}}$
$A > 2U_{ph\_r}^2$, $U_0 > U_1$	$\sqrt{\frac{A-B}{2}}$	$\sqrt{\frac{A+B}{2}}$

Here U_{ph_r} is a rated (nominal) supply line-to-neutral voltage RMS value,

$$A = \frac{U_{An}^2 + U_{Bn}^2 + U_{Cn}^2}{3},$$
$$B = \sqrt{\frac{1}{3}(2C - D)},$$
$$C = U_{An}^2 U_{Bn}^2 + U_{Bn}^2 U_{Cn}^2 + U_{Cn}^2 U_{An}^2,$$
$$D = U_{An}^4 + U_{Bn}^4 + U_{Cn}^4,$$

The cases with $U_0 > U_{ph}$ take place in three-phase three-wire circuits with asymmetric phase loads, which are having different kinds of the phase impedances.

The zero sequence phasor can also be expressed in polar form and consequences of it:

$$\dot{U}_0 = U_0 e^{j\alpha} = U_0 \cos\alpha + jU_0 \sin\alpha \qquad (18)$$

where, from (14),

$$\tan\alpha = \frac{\sqrt{3}(U_{Cn}^2 - U_{Bn}^2)}{2U_{An}^2 - U_{Bn}^2 - U_{Cn}^2} \qquad (19)$$

Here the use of (17) for U_0 with $U_{ph} = U_{ph\_r}$ is the most appropriate to position the phasor of the load voltages zero sequence component. This makes it possible to avoid calculations with the voltage RMS values raised to the power of four.

V. LOAD VOLTAGES ZERO SEQUENCE UNBALANCE FACTOR

For many years researchers and engineers uses not only the negative sequence unbalance factor magnitude, but also the static phasor of this unbalance factor, see, for example [19]. The corresponding phase data (the phase shift between the negative and positive sequence values) make it possible to compensate the unbalance of the related controllable value (voltage or current).

Here the static phasor of the zero sequence voltage unbalance factor is considered, which is based on the zero sequence and positive sequence voltage phasors relation:

$$\dot{K}_{0U} = \frac{\dot{U}_0}{\dot{U}_1} = \frac{\dot{U}_0}{U_{ph}} = \frac{U_0}{U_{ph}} e^{j\alpha} = K_{0U} e^{j\alpha}. \qquad (20)$$

According to (14), its barycentric and complex-plane representations are as follows:

$$\dot{K}_{0U} = -(u_n + v_n \dot{a}^2 + w_n \dot{a})$$
$$\dot{K}_{0U} = \frac{1}{3U_{ph}^2}\left[U_{An}^2 - \frac{1}{2}(U_{Bn}^2 + U_{Cn}^2) - j\frac{\sqrt{3}}{2}(U_{Bn}^2 - U_{Cn}^2)\right] \qquad (21)$$

The conventional zero sequence voltage unbalance factor is the modulus of the phasor:

$$K_{0U} = \sqrt{u_n^2 + v_n^2 + w_n^2 - u_n v_n - v_n w_n - w_n u_n}$$
$$K_{0U} = \sqrt{\frac{(U_{An}^*)^2 + (U_{Bn}^*)^2 + (U_{Cn}^*)^2}{3} - 1} \qquad (22)$$

where $U_X^* = \frac{U_X}{U_{ph}} \approx \frac{U_X}{U_{ph\_r}}$.

At last, the expressions, based on the refined values from Table I are adduced in Table II.

For the more representations of the phasors of the zero sequence load voltage and the respective unbalance factor in the barycentric coordinates and in the "new coordinates", please refer to [20].

VI. PHYSICAL INTERPRETATION OF NEUTRAL BIAS VOLTAGE PHASOR BARYCENTRIC COORDINATES

The neutral bias voltage phasor barycentric coordinates are, first of all, the coefficients of participation of the individual phase loads in this bias voltage generating. They can also be presented as the functions of the phase loads electrical admittances parameters.

Let's here assume that the three-phase star-connected circuit is the three-wire one and it has balanced supply, and let's for this mode reduce (1) to the form of (9).

After denoting $\dot{d}_X = \frac{\dot{y}_X}{\dot{y}_S}$, where $\dot{y}_S = \dot{y}_A + \dot{y}_B + \dot{y}_C$, $X \in \{A,B,C\}$, (1) looks as follows:

$$\dot{U}_{nN} = \dot{d}_A \dot{U}_{AN} + \dot{d}_B \dot{U}_{BN} + \dot{d}_C \dot{U}_{CN} \qquad (23)$$

Despite $\dot{d}_A + \dot{d}_B + \dot{d}_C = 1$, we should avoid the complex-valued coefficients and find the real numbers u_n, v_n and w_n.

Let's represent the admittances of the phases A, B, C loads in detail:

$$\dot{y}_A = G_A + jB_A = Y_A e^{-j\cdot\varphi_A} \qquad (24)$$

etc., here G_A and B_A are the phase A conductance and susceptance, respectively, $G_A = R_A / Z_A^2$, $B_A = -X_A / Z_A^2$.

The coefficients $\dot{d}_X$ can be written in algebraic form:

$$\dot{d}_X = d_{XRe} + j d_{XIm}, \quad (25)$$

TABLE II. ZERO SEQUENCE VOLTAGE UNBALANCE FACTOR

Condition and Mode	K_{0U}
$A < 2U_{ph\_r}^2$ $U_0 < U_1$	$\sqrt{\frac{A-B}{A+B}}$
$A = 2U_{ph\_r}^2$ $U_0 = U_1$	1
$A > 2U_{ph\_r}^2$ $U_0 > U_1$	$\sqrt{\frac{A+B}{A-B}}$

where

$$d_{XRe} = \frac{G_X G_S + B_X B_S}{Y_S^2},$$

$$d_{XIm} = \frac{B_X G_S - G_X B_S}{Y_S^2},$$

$$G_S = G_A + G_B + G_C,$$

$$B_S = B_A + B_B + B_C,$$

$$Y_S = \sqrt{G_S^2 + B_S^2},$$

Taking into account that for the case of the balanced three-phase supply

$$j\dot{U}_{AN} = \frac{1}{\sqrt{3}}(\dot{U}_{CN} - \dot{U}_{BN})$$

$$j\dot{U}_{BN} = \frac{1}{\sqrt{3}}(\dot{U}_{AN} - \dot{U}_{CN}), \quad (26)$$

$$j\dot{U}_{CN} = \frac{1}{\sqrt{3}}(\dot{U}_{BN} - \dot{U}_{AN})$$

the mathematical expressions determining the barycentric coordinates of the neutral bias voltage phasor can be reduced to the final form:

$$u_n = \frac{\left(G_A + \frac{B_B - B_C}{\sqrt{3}}\right)G_S + \left(B_A + \frac{G_C - G_B}{\sqrt{3}}\right)B_S}{Y_S^2}$$

$$v_n = \frac{\left(G_B + \frac{B_C - B_A}{\sqrt{3}}\right)G_S + \left(B_B + \frac{G_A - G_C}{\sqrt{3}}\right)B_S}{Y_S^2} \quad (27)$$

$$w_n = \frac{\left(G_C + \frac{B_A - B_B}{\sqrt{3}}\right)G_S + \left(B_C + \frac{G_B - G_A}{\sqrt{3}}\right)B_S}{Y_S^2}$$

Such the physical interpretation could serve to design the neutral bias voltage compensator, which stabilizes the barycentric coordinates (un,vn,wn) close to their desired values $(1/3, 1/3, 1/3)$.

The Mathcad and PSIM computer simulations have confirmed the validity both of equations systems (8), (27) and their full conformity with each other.

VII. CONCLUSIONS

The load voltages unbalance issues of the three-wire and four-wire three-phase star-connected circuits under the modes with unbalanced phase loads and the balanced sinusoidal supply (i.e. for the case of the absence of the negative sequence voltage component) are considered.

The complex-plane coordinates binding to the positive sequence component voltage static phasor has been implemented for the static phasor of the neutral bias voltage (and to the opposite to it zero sequence voltage vector), the static phasors of the load phase voltages and the static phasor of the zero sequence voltage unbalance factor.

The offered formulae require knowledge of just the phase loads voltages RMS values.

The positioning is proposed to utilize in load voltages unbalance compensation by means of a power electronics equipment control.

The static load physical interpretation of the coefficients of participation of the individual phase loads in the bias voltage generating (of the barycentric coordinates of the electric potential position of the three-phase load star common point) is shown through the conductances and the susceptances of the individual phases.

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